AISC 2018 (Suzhou)

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17–19 September 2018

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Chapter 1

17 September

1.1 Introduction: Jacques Fleuriot/Dongming Wang

DW stated that Suzhou was "The Heaven on Earth". He is from Beihang, which has main campuses in Beijing, but also other places, including Suzhou. (SUI-BUAA).

1.2 Automated Reasoning in the Age of the Internet: Alan Bundy

Noted that the Internet hosts a vast and growing amount of information. Most current queries are "factoid retrieval", with a lot of NL understanding to convert to formal queries. This is unambitious. What can we learn from what we know?

But challenges.

- Toomany potential axioms, drawn from a variety of sources, and we need to choose *dynamically* a small sbset on whcih to reasn.
- Some information is a depleted spate (RDF triples are weak — time, units etc. missing)
- diverse formats, therefore need *dynamic*[1] curation of information.

1.2.1 Guesstimation by Ontology and Related Techniques

A project that used Internet for facts: "how many golfballs does it take to encircle the earth", "what fraction of the UK would need to be covered with solar panels to meet the energy needs".

[1]Everyetime I see static projects, I wince.

1.2.2 Functional Reasoning Acquires New Knowledge (FRANK)

"What was the GDP in 2010 of the country predicted to have the largent population in Europe in 2018". We esitimate errors by error bars on numeric quantities. "What will the population of the UK be in 2025"? We have UK census data, so can extrapolate. FRANK uses alists as a flexible normal form for data. There is both uncertainyt of sources and uncertainty of inference methhods. For sources we look at various soures and analyse errors. Mthodlogy is simple. Look up fact. In this fails, decompose the query and recurse. Have both temporal and geospatial decomposition. E.g. "what will the population of Africa be in 2021"..

Alist is equivalent to reified RDF, but more accessible. Easily allows enrichment, e.g. with time and units. "How many cars doe sit take to create a traffic jam from Edinburgh to Glasgow", but looking up "length of car" just gives a number without units. Alist can also store uncertainty.

Use Hilbert's ϵ operator to slect the value that makes a relation true. FRANK can create regression objects, which can then be queried. Allows for "when will f exceeed g". Also integration, but this is tricky, e.g. integrating a population graph is meaningless.

Use Gaussian estimation, but via c.o.v $\frac{\sigma}{\mu}$ to handle scaling. Initialise ech source with high uncertainty, then undate by Bayesian inference. Posterior c.o.v estimates by comaring facts form different sources Favour hgh quality sources, e.g. World Bank. Some aggregation operations,e,g, regression, introduce errors.

1.2.3 Evaluation

Quite hard — not much related work. Odd, since when I first came into AI, query resolution was all about inference. Blames competitions and their emphasis on searc.

So we estimate accuracy by "leave one out" experiments. Here success = "within error bar", and this is 70–90%. Better than both Google Search and Wolfram Alpha, especially on prediction queries (70% versus 10 resp. 20). Our c.o.v. seems linearly correlated with actual error (when truth is knwon).

1.2.4 Enrichment

This is a big issue. Time often comes from context, e.g. "1911 census". Units are constrained by type length can't be in kg), and VW always reorts in mm (but never says so). GORT had a neat abduction of units. S ask the query, and the answers come back in clusers. Look at the ratio between clusters, and this will help fit imperial/metric. Surprisingly effective.

Huawie are interested in the question "what knowledge should we store locally" (e.g. o the 'phone). Want automatic partitioning of the question into local and remote components.

1.2.5 Q&A

CK One could design competitions for which this would be relevant?

A Yes, but difficult to evaluate. Hever, we should try — these competitions are very important.

DM What queries do you use for comparisons?

A Similar to those demonstrated: all requiring numbers See proceedings.

Q–JHD Try "what was population of Rome in 100AD", as this has two methodologies (area and corn) with very different results.

A Good example.

Q–CK What about using ML directly (neural networks) to generate regression?

A Not diretcly compared, but that is for exploration.

Q Geometry has "true on a component", so how does one handle this sort of thing.

A GORT looked at this. "Size of London buses" often returned answers for toys. "Diameter of golf ball" was wrong, because "everyone knows" the right answer (4cm), but we got 6cm ("golf ball light bulbs").

Q Ambiguity.

A Most common with NL queries, but currently not a major concern.

Q Biomedical imaging has very large datasets.

A

1.3 Concepts and Algorithms in Data Mining — Kotsireas

Data mining is about discovering patterns in data that cannot simply be discovered by queries. The aim is to "generalise" from the data.

Example 1 *Credit card companies use data mining (as the speaker discovered to his cost this summer).*

Example 2 *Online purchasing stores, e.g. Amazon.*

Decision trees = talked about ID3 — Iterative Dichotomizer. Looks for the "best decision", e.g. "is it an animal", or "is it my brother". The key concept is entropy. $-\sum_{i=1}^{n} p_i \log p_i$. Maximised when all $p_i = \frac{1}{n}$. Can extend to semi-spervised learningn with more examples.

1.3.1 Associatio Rules

Purchase–Basket model. Association Rule is $(x_1, \ldots, x_n) \Rightarrow (y_1, \ldots y_m)$.

- Support($X \Rightarrow Y$) = fraction of elements in the dataset that contain all of $x \cup Y$.
- Confidence($X \Rightarrow Y$)=$\dfrac{\text{Support}(X \Rightarrow Y)}{\text{Support}(X)}$.

Then we want good ARs.

1. Generate all itemsets with support exceeding a given threshold: "large" itemsets.
2. Generate rules for each large itemsets.

Step 1 has exponential complexity. But we have "downward closure" — a subset of a large itemset must be large. Also "anti-monotonicity": a superset of a small itemset is small. Hence an iterative algorithm, starting with 1-itemsets.

1.3.2 Q&A

Q Quantum recommender systems.

A I've not seen a quantum computer.

1.4 Into the Infinite: Theory exploration for co-induction

Automatically coming up with verified properties of programs. HIPSTER is such a system for Isabelle/HOL.

```
List A = NIL  Cons a (List a)
```

Induction only applies finite lists, but co-induction will support infinite lists as well. Live demonstration of `cohipster`, which can be imported into Isabelle. One proof actuallyused a previously discovered lemma.

The Isabelle theory is translated to Haskell, pre-processed, the QuickSpec tool looks for equational properties of Haskell programs. Gneerates terms, instantiates with arbitrary values and checks for evaluation equality. Then conjectures equality. Then back-translate as conjectures. Pass to "routine reasoning": if true discard (as trivial), and if false discard. Pass rest to "hard reasoning": inductive or co-inductive reasoning. Those that are proved are offered to user. Those that can't be proved are tried again (proofs based on previous lemmas), and those that are left are offered to the user as interesting conjectures.

But we need to use observational equivalence for QuickSpec on infinite objects. These observers are automatically generated, but there are tricky aspects (don't want to explore large binary trees for example). Subgoal proofs in "Hard reasoning" via Sledgehammer. For Hintze's Stream Calculus, we discovered 9 out of his 18 rules. 3 of these were out of scope (conditional, lambda-expressions) so 9/15-60%. We also found more lemmas (precision 21%).

1.4.1 Q&A

Q Hard versus Easy?

A "Easy" is simplification, rewriting and first-order logic. (Co-)induction and Sledgehammer are "Hard".

Q How did you generate infinite objects?

A JHD didn't follow the details.

1.5 What does qualitative spatial reasoning tell us about Origami? Ghourabi

Examples in satellite antennas and DNA origami (DNA boxes). Note that we can construct, for example, the regular heptagon. Note that Origami's (O6) solves cubics.

Qualitative spatial reasoning. Use RCC-calculus. Relations such as equal, disjoint, proper-part etc. Used Matheatics to definie the systems S of algebraic constrants that describe the fold line. Then compute a GB, and then discriminants. Then the hard problem is giving these geometric interpretations. For example (O5) gives 0/1/2 fault lines depending on distances. Similarly for (O6).

This gives us an intuitive representatiion using qualitative spatial calculus. Aim to take this to a formalisation is Isabelle or Coq.

1.5.1 Q&A

Q Is Origami an "efficient" method fro solving equations.

A We aren't really concerned with this, but a good question.

Q Could this lead to generating Origami from a 3D shape?

A Yes, there are reverse-engineering approaches.

Q In ruler+compass, we know what is constructible.

A We have similar results, basically adjoin the cubic..

1.6 Automated Formailization using Statistical and Neural Approaches: Kaliszyk

AI can win in easy (and hard) games, and we feel this can be extrapolated. Some higher math is too hard for humans. Computer-assisted proofs; Kepler, 4-colours, Feit–Thompson.

Qprobems Alphhazero uses sellf-play to learn.

Repeated strategy improvements.

Wider search of the game tree.

Take 11000 statements from Flyspeck. 72 overloaded instances of "+" etc.

Seed an RNN with a sequence of tokens. Example of machine translation. This is the training phase, then we have evaluation. Claims that "attention" is the key element of machine recognition of mathematics. Mizar is "only" 50k statements, so we lookat the intermediate statements in the journal, to get more training data. Of we allow "at most two errors" we get to 90% accuracy.

As with natural language, translating a variety of incompatible things (different natural languages) actually improves the translation.

1.6.1 Q&A

Q What about HOL etc.

A Incompatible extensions etc. Mizar has a committee which ensures unique extensions etc. But this is a big effort. Makes Mizar the best system for this sort of machine learning.

1.7

Claims that software development is very error-prone. But this is also true of CAS, especially Mathematica. Cites [DPV14].

Hence my PhD thesis is formalisation of linear algebra systems. We want our results to be both verified and usable in practice. Based on the HOL Analysis section. Represent a vector by a selection function. But need a projection from the abstract representation to a concrete representation in terms of immutable arrays. This handles many things, but for Hermite Normal Forms, , there is no standard definition: some go by row and some by column, some restrict to square and some to nonsingular. Need the concepts of a compete sets of nonassociates. Similarly a complete set of residues. With these, we can produce a geeral definition.

1.7.1 Q&A

Q You were worried about ML. Is this so you can export to Haskell?

A Yes.

Q Isabelle doesn't necessarily care about computability.

A HOL Analysis Library is OK.

Q ?? computed with Axiom, but got a lot of $f \neq 0$ side-conditions. Does you method help exclude these?

A Might do.

Q Performance — you mention machine ints, but what about floats?

A We don't use these – computer algebra

1.8 The Accessibility of Mathematical Formulae for the Visually Impaired in China: Su

Braille and speech are the two means for the visually impaired. We have a Web 2.0 platform that translates Chinese into Braille and speech. Chinese Work Segmentation is a challenge (there are no spaces between "words"). Use an LSTM+RNN algorithm for this. But there is a shortage of corpus for Braille segmentation, which is different from the usual (?).

There's also a Polyphone (one character has several pronunciations) problem, which can also be hard for humans.

Specialist translation is not new. For Chemistry, we have a ChemML to Braille translation. This is in Chem4word. Also MusicXML, to the special Chinese Braille Musical Standard (allows for traditional Chinese instruments).

So for mathematics there is MathML. In Braille we have Nemeth and Marburgm which differ, but Chinese Braiile is neither. They differ in both themapping and the sequence of characters (some have denominator before numerator, for example). Shows it in Word, but note that Miccrosoft's entry mechanism uses themouse, which is very hard for the impaired. So we have our own input mechanism.

Speech is also a challenge.

1.8.1 Q&A

Q This looks pretty bewildering.

A Indeed. Not published yet.

1.9 Machne learning for inductive theorem proving: Jiang

The Boer–Moore model is designed fro inductive theom proving. See ACL2, and [Boulton1992] in HOL88. Boyer–Moore is basically waterfall. e pour the goal over pile of heuristics thatth]ry to solve or simplify, an anyting remaiing fallsinto the ool. We include al the HOL lIght auotmation tools. For the subgoals in the poolm we use indction. Problem in to choose theinducton variable.

On the other hand, there ae Hammer systems. Lemma selection (machine learning) etc. So can we merge the two? HOL(y) Hammer selects keywords from the statement. End up with three diffrent waterfalls: HOL(y), Simplify+MESOn and straight (several inductive statements are proved by straight induction). This multi-waterfall does better than any previous, but the Venn diagram seems tobe completely populated.

```
DROP (a+b) l = DROP a (DROP b l)
```

is a good example.

1.10 Formalising Some "Small" Finite Models of Projective Geometry in Coq: Braun

- There is always a line through two points
- For all lines, there are at least two points
- There are noncollinear points.

The synthetic geometry appproach is close to the visual description, and is short and effective, But requires a linear variety for each dimension (point, line plane space) Needs a simplification method for each dimension.

Suggests a rank function as an answer: $\mathrm{rank}(\{A, B, C\}) = 1$ is "A, B, C collinear".

Theorem 1 *The two are equivakent, in 2D, $> 3D$ and 3D.*

There are proof techniques: equality test, closure of assumptions, contradicion to eliminate degerate cases, etc. Butthere's a meta-problem of scheduling these.

Desargues theorem has 10point and 10 lines, therefore 20 quantifiers. Hence case analysis is difficult. Management of non-degeneracy is difficult. He also finds that Coq memeory usage is a major bottleneck. Open problems include Hall planes of order 9. Would be interested in SAT/SMT soling as well.

1.10.1 Q&A

Q You said "the proof is not available", but isn't it there?

A Not in a form we can understand

1.11 FMUS2

Want to find a minimal unsat core. Research problem is to enumerate all MUS. There are two proposed algorithms

- Complete MUS Enumeration Algorithm
- Partial MUS Enumeration Algorithm. Producdes first quickly, then egenated the rest incrementally.

We propose a new FMUS2 algorithm This will work for edcidable fragments of FOL, not just SAT. We consider FEF (function-free, equality-free fragment of FOL). Then concept of MGU. $L_1\sigma = L_2\sigma$ and any other such is $\omega \circ \sigma$. Note that sometime we repeated answers when trying hard instances.

1.11.1 Q&A

Q Some fragments of FOL can handle equality axiomatically.

A We can't handle this yet.

1.12 Discovering geometry theores in regular poygons: Z. Kovács

Two diagonals determine an intersection point. What is the distance of two such points? Show that in a regular n-gon, he only rational lengths are 1 and 2, and the only quadratic such is $\sqrt{3}$. Note that 11-gon is not constructible with ruler-and-compass or by origami. We use Wu's aporoach for algebraizing the setup, GBs, [WatkinsZeitlin1993] based on Chebyshev polynomials to describe rotations by $2\pi/n$, so get the minimal polynomials of $\cos(2\pi/n)$.

Example 3 *In a pentagon there's a $\frac{3-\sqrt{5}}{2}$ length between such points. But ths doesn't follow from the hypotheses, because of the star-regular pentagon, in which it's not true. Also, we need a quartic, because $-\frac{3-\sqrt{5}}{2}$ is also legitimate.*

Note that a regular n-gon has $\left(\begin{array}{c}\left(\begin{array}{c}\left(\begin{array}{c}n\\2\end{array}\right)\\2\end{array}\right)\\2\end{array}\right)$ choices. Example of a case visualised in Geogebra.

Also there's an example in the 11-gon where what looks concurrent is actually not, by $0.04\ldots$.

Chapter 2

18 September

2.1 An Exploration to non-NN style deep learning: Z.-H. Zhou

2.1.1 Q&A

SIAM: “Deep Learning is ML with deep Neural Networks”. NN are not new. [Our model of] neurons is very simple: and we can study its mathematical properties. Number of layers has increased from 1–2 to 8 in 2012 and now thousands. The activation functions is continuous and differentiable, usually a sigmoid. Back propagation relies on this. A serious question is why we need deep networks.

If we increase the model complexity we can increase the learning ability. We can add units (model width) or layers (model depth). Adding layers increases complexity more. But increased complexity causes increased risk of overfitting (and difficuty of training). The solutions are big traning data, and powerful computing facilities. Error gradient can diverge when propagated in many layers, difficult to converge to a stable state, hence difficult to use cassical BP. Many tricks to solve this.

But why deep? In 1989 is was proved that one hidden layer was theoretically sufficient. But in practice deeper does better. These days we do not use manually-designed feature design, but the system learns feature — “representation learning”. A major text [GoodfellowBengioCourville2016] claims that the first hidden layer finds edges, the second corners, etc., though this is an idealisation. Compare with decision trees, which always use the original features. Boosting has the same problem — it can’t learn new features.

A major problem is the number of hyper-parameters, so when people say “CNN” they are actually using different models. Note that many winners in Kaggle competitions are random forest or other traditional ML technologies. I observe that Deep NN work well for numeric, but not those with discrete features. Why?

We propose gcForest (multi-grained Cascade Forest). Many fewer hyper-parameters. Heavily based on ensemble learning A lot of KDDcup winnes have been ensemble-based. Look at Error-ambiguity decomposition: $E - \overline{E} - \overline{A}$ where $\overline{E}$ is the accuracy of the individual models and $\overline{A}$ is the diversity of the models. Note that you can't optimise both of them. But there's no operational definition of "diversity". See my boo on Ensemble Methods. This is similar to "Stacking", but that was very subject to overfitting with > 2 layers. Random forest randomly slected $\sqrt{d}$ features, then chooses the best, whereas completely random forest just takes a single feature at random. Generally does better, especially on smaller datasets. Note that these models are not differentiable, which responds to a challenge from [Hinton2015]. Note we are not claimng this is the answer to all problems — image data responds well to DNNs.

We are collaborating with Essence Securities, company behind China's mpayments industry. Their real challenge in detection of illegal cash-out transactions. Trainged on 131M instances, with 171K positive. Tested on 52M/66K. Note there are more than 5000 features/transaction. Most o fthese are symbolic. Under the academic measures (AUC, F1, KSD) we do better than LR, DNN and MART, also with industrial performance figures (rates of transaction decline).

2.1.2 Q&A

Q Cost?

A Generally cheaper than DNN in terms of operations, but we don't get the GPU gain that DNNs typically have. Our industry partner uses distributed processing for this. We think Intel's KNL may be very suitable for gcForest. We have an Intel centre looking at this.

Q Hyperparameters — you seem to have fewer than many forest models.

A True — we could add more. But still many fewer than DNN. But there would be a great computational cost

Q Learning the hyper-parameters?

A This is "auto ML". Many people like this but I am pessimistic. I think a successful auto-ML would show P=NP.

2.2 Towards Intelligent Mathematical Documents: Xiaoyu Chen

Quotes Wikipedia's definition of a documentation as a "representation of thought". Carvings, parchment, paper (writing, then printing) and now electronic.For an "intelligent document" we are looking at communicatoinn of thought between humans *and machines*. Example of an "intelligent book" in biology. But representation is a problem: talks about OpenMath/OMDoc as an XML-based solution. But Isabelle, Coq etc. have different formats, and different semantics. Example of dynamic geometry, which also have an interchange requirement hence InterGeo.

Need a fine-grained structure [KMW04]. Encapsulations + Relations gives us Geontology. Shows Chinese and English versions of same text.

For mathematical documents, formula searching is a major issue. If we linearise, we can use text-based methods (MIaS). Or tree-based methods. Need transfroms from natural language to our GDI, then processing and (natural language) output.

How do we recognise documents, e.g. a diagram in which Simson's theorem is implied. First image processing tools, then recognise basic geometric relationships. Then can try to deduce what the theorem is, but for Simson's figure there are actually two: Simson's Theorem and its converse.

2.2.1 Q&A

Q Why geometry?

A It's rich (diagrams, dynamic tools), and we have good formal deduction techniques.

Q OMDoc can import, but not export.

A Only as LaTeX.

Q Level of detail? Beginner/expert.

A Rather like intelligent tutoring systems. Need a knowledge model. But we haven't considered how to do the actual teaching yet.

2.3 Automatic Deduction in an AI Geometry Book: Quaresma

Therefore Open Geometry Prover — an open source project we are attempting to revive. `https://github.com/ivan-z-petrovic/open-geo-prover`. We want libraries. We have Wu's method, and (with problems) the Area Method. Initialideas on Full-Angle method.

Currently have a library of 236 problems proved with TPTP. With this, we have four xsd files, 12GATP container is an extension of the I2G container. Various proof systems: Geogebra etc, Ciinderella (statistical), GeoProof (Coq).

History of Coimbra and its Geometry tiles.

2.3.1 Q&A

Q Your title was 2D.

A For the moment, yes. But there is nothing fundamental here.

Q Intergeo was largely a European project.

A It was a bagful of cats. I am not sure if the cats are still angry, but I am not trying to corrall the DGSes.

2.4 Towards an Automated Geometer: Z. Kovács

I am in Geogebra development. Much pressure to develop Geogebra. Good survey of previous work. Also use the GIAC implementation of Gröbner bases. WebAssembly is the way of the future for doing high-performance computation in the browser.

Example 4 (9-point circle; [Briançon–Poncelet1821, Feuerbach1822]) *Showed* `ProveDetails` *in Geogebra. Note that the syntax only allows four points. Some quadruples work, others produce degeneracy side-conditions.*

2.5 JHD Tutorial

See `http://staff.bath.ac.uk/masjhd/Slides/AISC2018-JHD.pdf` for the slides, or [Dav18].

Chapter 3

19 September

3.1 Find-Path: A Paradigmatic Problem for AI and SC: Chee Yap

3.1.1 AI

Is AI Symbolic or Numeric? Obviously SYmbolic, but all the ML stuff is numeric. Showed a Minsky diagram "what is an apple" — symbolic description or numeric weights on a graph. See also Rod Brooks "Elephants don't play Chess". Also showed [DoranMitchie1966ProcRoySoc].

Old joke "the spirit is willing but the flesh is weak": went English Kannada Chinese Tajik English and got Rubbish. "Invisible imbecile" became "the ears cannot see the eyes". Notes the EU large spend on MT, but has humans in the loop for high-quaity MT. Claims [François Chollet] MT is "congitive prosthetics".

3.1.2 SC

Claims "exactness", but really "error-free" is the goal. Again we can ask "symbolic or numeric", and in fact there's a surprising amount of numerics, e.g. the meaning of $\sqrt{2} \in [1, 2]$". Pretty much all applications need numerics. Example of real root finding.

Zero problem is real, but one-sided (if non-zero, we will find it).

3.1.3 Robotics

Ai has always seen robotics as its special province. [Brooks1983: Solving the Find-Path Problem by Good Representation of Free Space]. Yap's work can be seen as making this rigorous. Here $f : (A, B.map) \mapsto Path$. See [SS83, SEDS17] for symbolic methods: very expensive. [Brooks1983] claims sampling/subdivision works. Claims this is correct, but we need to address fear of numerics.

3.1.4 Soft Subdivision Search

Note the concept of "configuration space" $\subset \mathbf{R}^d$, which depends on the robot. Let $\gamma \in \mathbf{R}^d$. FPrint$(\gamma) \subset \mathbf{R}^k$. $f : (\Omega \subset \mathbf{R}^k, B_o \subset Cspace, alpha, beta \in Cspace) \mapsto path \cup failed$.

- Exact Approach (gold standard) obeys P and N
- Subdivsion Approach: RP: if there is a path and the resolution is fine enough, will find it.
- Sample Approach. Might find it.

Many people try to fix RP by saying "is there a path of clearance ϵ".

I want $\exists K > 1$, where we return a path if one exists of tolerance $\geq K\epsilon$, and returns `failed`, if there isn't one of tolerance $\geq \epsilon/K$. We work by subdivision in $\mathbf{R}^d$: a box is Free/Stuck, Mixed or ϵ. But note that ϵ is in physical space, not $\mathbf{R}^d$. Note how we're very dependent on union/find data structure. Want a conservative evaluation on boxes.

The community uses `double` so we do, even though we really only have rigour with bigfloats. Can get as far as 5DF (Ring robot).

This is a case, like JHD's talk, where we need to put the AI back into SC algorithms.

3.2 Revealing Bistability: (Changbo Chen and) Wenyan Wu

Interested in biological neural networks. Need to consider fold and Hopf bifurcations. $\dot{x} = F(x, u)$ where $x = (x_1, \ldots, x_m)$ are variables and $u = (u_1, u_2)$. Fold: J_F has a zero eigenvalue, Hopf: J_F has a pair of pure imaginary eigenvalues. Only interested in $u_i > 0$. Cites various symbolic approaches, including [BDE$^+$17].

$F = 0$, $J\mu = -\omega v$ $J\nu = \omega\mu$, $\alpha\mu - \beta\nu = 1$, $\alpha\nu + \beta\nu = 1$. Then this system defines both types of bifurcation, $\omega = 0$ being fold. Let B_H be the stability boundary, a projection of this set. Assume

1. $V_{\mathbf{R}}(P_f \cap \pi^{-1}(R)$ is compact
2. $\dim(V_{\mathbf{R}}(P_H)) = 1$
3. At each regular point of $V_{\mathbf{R}}(P_H)$, the Jacobian of P_H has full rank.

Proposition 1 *Then $R \setminus B_H$ is divided into finitely many cells.*

Shows an algorithm for an approximate stability boundary.

Alzheimer's disease. trace$(J_F) = -k_1 - k_2$, so no Hopf bifurcations. $x(x_1, x_2)$ but many parameters. Guess parameters, solve, and get some rescaling hints.

Get a region between two curves which has bistability. For K_a get a boundary $[109.\dots, 501.\dots]$ but [DeCaluwe2013] had $[105, 520]$ by numerical evaluations.

$M\zeta$. Three variables. Again need a sensible rescaling. There are also some "false" boundary curves (projections of equilibria with negative coordinates). Equilibrium with low concentrations, but disappears when $s > 0.01$, and then stuck in the high state.

Use numerical homotopy to do the equivalent of projection.

3.2.1 Q&A

Q Formulae?

A Not directly — numerics from homotopy.

3.3 Early ending in Homotopy Path-Tracking for Real Roots: Yu Wang

Mtivated by finding sample points on real components.

Example 5 $f = (x^2 - x^3 + 4x + 1) \cdot ((x - y_6)^3 + x + y) = 0$. *Three real components, one of which is a small closed curve.*

Assume the Jacobian has generic full rank. Get a square system to solve. These are sparse, so we look at Hom4PS (2.0). But it could be 25×25, whihc is challenging. The real root bound is overkill, so what do we do?

"Good" is isolated ....

This is implemented in LHP. Since much source code is proprietary, I can't compare. Might lose real roots if $x(t)$ is steep.

3.3.1 Q&A

Q Can you quantify "steep"?

A Numerically.

3.4 Autocorrelation via runs: Jing Yang

We have a sequence $a_i \in \{+1, -1\}$. Interested in $k - runs$ of identical values. The Periodic Autocorrelation Function (RAF) is $p_k := PAF(A, K) = \sum_{i=1}^{n} a_i, a_{i+k}$ where $i + k$ is interpreted mod n. Preserved by rotations. Hence assume A starts with $+1$ and ends -1. Also assume $s = \sum a_i$ is known.

Hence PAF Profile Problem (PPP): given the PAF, find the sequence.

Example 6 $n - 9, s = -1$ *and PAF* $(-3, 1, -3, 1)$. *The PAF are quadratic constraints.*

This is motivated by the Hadamard maximal determinant problem: Largest $\nu \times \nu$ determinant $\in \{+1, -1\}$. This is still an open problem.

Consider the partition of runs in A. Then solving a PPP is equivalent to finding the runs, which is an ordered partition of n. The number of runs is $P_0 - p_1)/2$, and the number of 1-runs is $(p_0 + p_2 - 2p_1)/4$. In our example, we have 6 runs, and 4 one-runs. Hence $1^4, 2, 3$ are the run lengths. s will tell us which of the long runs are positive/negative (in this simple case), so the positive runs are $(1, 1, 2)$, which has three possible orders.

Hence an algorithms, looks to JHD like sequential search among so-constrained options. There's a step to remove certain rotations, but apparently only even-length ones. Modulo rotations, there is only one version of $(1, 1, 2)$. Then need to interdigitate all permutations of $(1, 1, 3)$.

Theorem 2 *[Ehlich1964] If R_1 and R_2 are circulant (± 1) matrices of order n and ..., then $H = \begin{pmatrix} R_1 & R_2 \\ -R_2 & R_1 \end{pmatrix}$ has maximal determinant.*

Get an example $2^{161}3^4 5^{40}$ in a few hours of C (much faster than Maple).

3.5 Game-theoretic analysis on the Number of Participants in the Software Crowdsourcing Content: Peng

Look at "winner takes all" methods in "all-pay auction model".

Any player who has joined the contest is called a "candidate". Expected number of participants is $\sum_i p_i(join)$. This should be a Nash equilibrium: C_i's expected payoff with $p(\sigma_1, \ldots, \sigma_{i-1}, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n) \geq p(\sigma_1, \ldots, \sigma_{i-1}, \sigma_i', \sigma_{i+1}, \ldots, \sigma_n)$ for any other strategies σ_i'.

3.6 Speciality-Aware Task Assignments: Song

Example: Uber tasking. So we are assigning offline spatial tasks to online workers. But, e.g. Party Organisation, may require specialist skills. Some tasks may require more than one person, and several skills. Different charging rates also.

A set of task requestors T, each task has a location l_t, skil set + budget; a set of workers with .... can prove thatthe allocation is NP-hard. First try a greedy algorithm — greatest total budget first. Assign worker minimising $\frac{r(w,t)}{|s_t \cap sw|}$. Alternatively task with greatest budget/#tasks. We had a real data set, $|T| = 500$, $|W| = 5000$ from CSTO, but we had to generate locations. The two algorithms are very similar, both much better than the baseline.

3.6.1 Q&A

Q Did you try any exact solutions, e.g. MathSAT?

A No.

3.7 Chinese New Word Detection Approach Based on Independence Testing: Jiang

New words keep happpening (e.g. Internet). But written Chinese has no word boundaries. There are no morphological rules and no general rules for new word construction. So "word out-of-vocabulary" is a major problem for Chinese processing. Various methods proposed, but often training-corpus dependent. Also subjective choce of parameters.

Defines "Neologism" (Wikipedia). Definition of a word is that it can be used independently. Statistically

- The characters in a new word should be correlated
- Should show a certain independence and flexibility
- The word should appear in the text at a certain frequency so people can recognise it from context.

A seminat unit is aChinese character strig that can represent a relatively complete meaning. For a given text T, the string of two adjacent semantic units is a semantic pair. Claims that being first and second should be independent for the independence hypothesis above. We set a basic fequency threshold. We use Heaps' Law: if there are N words in a text, there shuld be kN^β different words. In examples $\left(\frac{N}{2}\right)^{0.25}$.

Example 7 (Trump) *Three symbols.*

Example 8 (But) *"People" and "Civilian Run" are each two symbols, with the second of "People" being the first of the other. So should we create a new work here out of the three characters, as above?*

Tried on People's Daily (1.8M characters), and 100 downloaded random news articles. The precisions were 81% and 91%.

3.7.1 Q&A

Q Is it the case that all words are made from existing pictograms?

A Yes.

3.8 LaText: A Linear Algebra Textbook System: Shuai

Joint Xiaoyu Chen, Dongmig Wang etc.

Textbooks play a fundamental role in education and knowledge dissemination. But a "textbook in a database" is not sufficient.

LA is fundamental, so many potential users. Also can benefit from powerful computing support. We work with *knowledge objects*, manipulable semantic units. Belief = Type+ID+Role, and a HasText relation to Text=ID+Format+Content+Language. A Belief may have a Solutioon (proof).

Implementation is based on Angular. Interfaces on desktop and mobile. There is autogeneration (contents, menu, index) from the graph database (neo4j or ...).

3.8.1 Q&A

Q What is the role of examples? Can one add negative examples.

A Just add examples.

Q Why graph database rather than RDF triple stores?

A I believe speed.

3.9 Closing

Dates JHD mentioned clashes.

CK What about competitions and data sets. These stimulate research.

AB Needs selection and maintenance of the aim (JHD's words!). Noted that #topics is #people in room.

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