Corrections for *Ordinary Differential Equations* by H. Logemann and E.P. Ryan

p. 13: error in Example 1.1. The initial-value problem considered is

$$\dot{x}(t) = tx^{1/3}(t), \quad x(0) = 0. \tag{$*$}$$

In the form stated, the example is not correct: more precisely, the function $x : \mathbb{R} \to \mathbb{R}$ given by

$$x(t) = \begin{cases} \big((t-c)/\sqrt{3}\big)^3, & t \geq c \\ 0, & t < c, \end{cases} \qquad \text{where } c \geq 0,$$

is not a solution of $(*)$ whenever $c > 0$.

Correction. Each of the following three functions $\mathbb{R} \to \mathbb{R}$

$$t \mapsto 0, \qquad t \mapsto \big(t/\sqrt{3}\big)^3 \qquad \text{and} \qquad t \mapsto \begin{cases} \big(t/\sqrt{3}\big)^3, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

solves the initial-value problem $(*)$. Moreover, if $(*)$ is replaced by

$$\dot{x}(t) = x^{1/3}(t), \quad x(0) = 0, \tag{$**$}$$

then, for every $c \geq 0$, the function $x : \mathbb{R} \to \mathbb{R}$ given by

$$x(t) := \begin{cases} \big(2(t-c)/3\big)^{3/2}, & t \geq c \\ 0, & t < c \end{cases}$$

is a solution to the initial-value problem $(**)$.

p. 113: error in the proof of Theorem 4.11. It is concluded from the supposition $\sigma = \omega$ that $x(t) \in C$ for all $t \in [0, \omega)$. This is not correct. What can be concluded is that there exists a sequence $(t_n)_{n\in\mathbb{N}}$ in $[0, \omega)$ such that $t_n \to \omega$ as $n \to \infty$ and $x(t_n) \in C$ for all $n \in \mathbb{N}$.

Correction. Seeking a contradiction, suppose that $\sigma = \omega$. Then there exists a sequence $(t_n)_{n\in\mathbb{N}}$ in $[0, \omega)$ such that $t_n \to \omega$ as $n \to \infty$ and $x(t_n) \in C$ for all $n \in \mathbb{N}$. By compactness of C and openness of G, there exists $\varepsilon > 0$ such

$$C_\varepsilon := \{z \in \mathbb{R}^N : \operatorname{dist}(z, C) \leq \varepsilon\} \subset G.$$

The set C_ε is compact and so, for all $n \in \mathbb{N}$,

$$S_n := \{s \in [t_n, \omega) : x(s) \notin C_\varepsilon\} \neq \emptyset,$$

because otherwise the closure of the set $\{x(t) : 0 \leq t < \omega\}$ would be compact and contained in G, and thus, by Corollary 4.10, $\omega \in I$, which is impossible. Defining the sequence $(s_n)_{n\in\mathbb{N}}$ by $s_n := \inf S_n$, it is clear that $s_n \to \omega$ as $n \to \infty$, $\operatorname{dist}(x(s_n), C_\varepsilon) = \varepsilon$ and $x(t) \in C_\varepsilon$ for all $t \in [t_n, s_n]$. Setting $B := \max\{\|f(t,z)\| : t \in [\tau, \omega],\ z \in C_\varepsilon\}$, we have that

$$\varepsilon \leq \|x(s_n) - x(t_n)\| \leq \int_{t_n}^{s_n} \|f(t, x(t)\| \mathrm{d}t \leq B(s_n - t_n) \to 0 \ \text{ as } n \to \infty,$$

yielding the desired contradiction.

p. 211, 2nd line below equation (5.39): typo. "$[0,t] \in T_2$" should read "$[0,t] \subset T_2$".