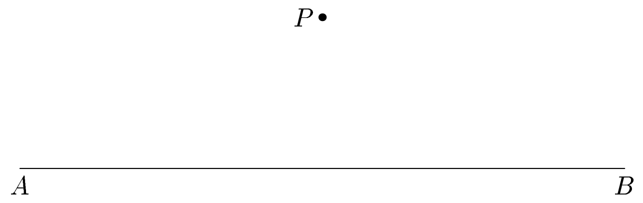
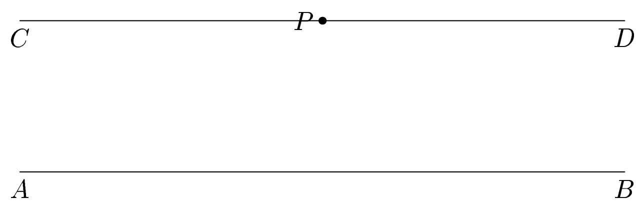
What's the problem?



We want to draw a line through P, parallel to the line AB. Euclid thought we could do it. He also thought there was only one way of doing it.

For many years people tried to *prove* that Euclid was right, but nobody succeeded.

In the end people gave up, and simply assumed that he was right.



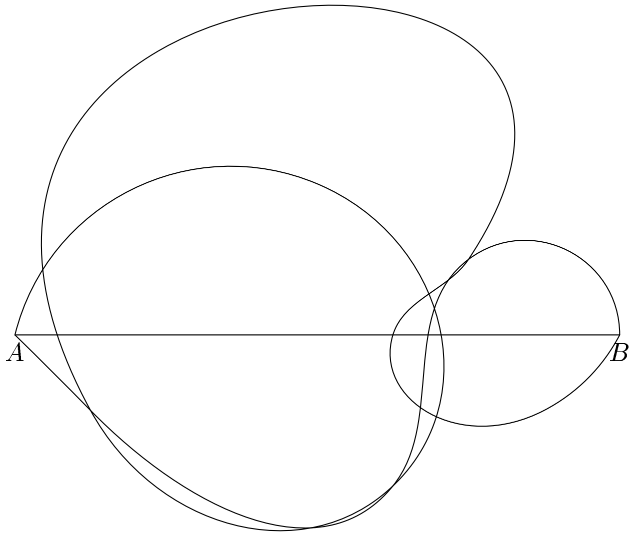
You can tell that the line CD either doesn't meet AB at all, or, if it does, it meets it on both sides of P.

But how do you know that AB doesn't meet CD on both sides of P?

Or how do you know that you can't wiggle CD a bit, and still get something which is parallel to AB?

What is a line anyway?

If we are going to answer questions like these, we had better know what a line is in the first place.



A line is the shortest route between two points. That is why Roman roads go in straight lines.

This is a good way to define a line, because we can test whether it's true or not. So we can recognise a line when we see one.

Euclid's geometry was developed from practical geometry discovered by the Egyptians. They lived in a flat plain. In a plain, the shortest route between two points is the straightest one.

It's not always obvious what the shortest route is. If you want to go to America, you don't fly West. So if you were living on a rather small ball you might have a different idea of what a line is.

Short paths

Imagine a village between two hills. To get from one place to another, the villagers won't climb over the hill and walk back down again. Instead they will walk round the hills.

If they do this a lot, they will make some paths. The paths will be the shortest paths they can make. An eagle, looking down on the village, might see something like this:

Boat lines

Here are a few islands in the ocean.

One of the islanders decides to start a boat service among the islands. She wants her boat journeys to be as short as possible. She can only let passengers on and off at islands. What lines will she be interested in?

She will only be interested in lines like these, which pass two or more than two islands.

If we have a “line” AB on the ball and a point P not on it, there will be no “lines” through P which don’t meet AB. A “line” is really a big circle, whose centre is the centre of the ball, and any two of those circles will meet. Any triangle will have angles whose sum is more than 180°.

The eagle looking down on the village in the mountains sees a path AB and a point P, and lots of other paths through P which don’t meet AB. Any triangle will have angles whose sum is less than 180°.