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It's also true that if $p \neq mn$ then p is prime, but that's slightly harder: let's not bother about it.

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If I allowed 1 as a prime, that wouldn't be true, because $6 = 2 \times 3 = 1 \times 2 \times 3$. So we'll agree that $6 = 2 \times 3$, and instead of arguing about how many 1s to write, we'll decide not to write any.

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You need chocolate. . .

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Primes get rarer and rarer as the numbers get bigger.

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What else?

5 is prime;

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5 is prime; so is $5 + 6 = 11$

What else?

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And we don't know lots of other things. . .