Prime Numbers I for Helpers

A. 5 *and* 7 *are prime; so are* 11 *and* 13*, and* 17 *and* 19*. Find two prime numbers bigger than* 100*, one of which is* 2 *more than the other.*
101 and 103 are the first such pair.
Is it possible to find two primes, bigger than 10*, one of which is* 3 *more than the other?*
No! One of them would be even.
4 *more?*
Yes - 13 and 17, for instance.
Is it possible to find three primes, bigger than 10*, such that the second is* 2 *more than the first, and the third* 2 *more than the second?*
No! one of them would be divisible by 3.

B. How many prime numbers are there between 100 *and* 200*? It will be easier if several of you work together on this one. Try to do it without a calculator.*
They should break it up into sections: one group of them can look at 101–130, another at 131–160, a third at 161–190, and they can all have a go at the 190s. They only need to think about numbers ending in 1, 3, 7 and 9, and if you like you can tell them how to test quickly for divisibility by 3. They know $11^2 = 121$ but they might not spot $7 \times 17 = 119$, $7 \times 19 = 133$, $11 \times 13 = 143$, $7 \times 23 = 161$, $13^2 = 169$, $11 \times 17 = 187$.
There are twenty-one, namely 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197 and 199.
Do you think there will be as many prime numbers between 1100 *and* 1200 *as there are between* 100 *and* 200*?*
No: primes thin out, because there are more and more smaller primes that could be factors of big numbers.

C. Try adding 1 *to square numbers. Do you ever get a multiple of* 3*? Of* 5*? Of* 7*? Of* 11*? Of* 13*? Of* 17*? What if you subtract* 1 *instead?*
They should find that adding 1 to a square can give a multiple of 5 or 13 or 17, but not of 3, 7 or 11, and that subtracting 1 it's the other way round.
This is an extra question for if they run out of things to do.