Exam feedback on MA40188 Algebraic Curves

Nobody scored 100% (see below for why not) but several people got quite close and apart from a very few who had not troubled to learn even the most basic things, everybody did all right.

Q1. The very few identified themselves immediately by not being able to answer part (a) of this question. Non ragionam di lor, ma guarda, e passa. The only other way of dropping marks hereabouts was to forget to say how K^* acts both here and in part (b), but nearly everybody gave a full answer to the two parts taken together. Part (c), which I did in the lectures in two different ways and is in any case easy, caused only very little trouble but shouldn't really have caused any. Some answers were not integers, which is unimpressive. There are several different ways of answering (d) and almost everybody gave at least one of the correct ones, and (e) was well known. At part (f) a few more people started to lose the plot, typically confusing "homogeneous polynomial" with "homogeneous coordinate" or not answering the question about why the definition makes sense, and a few more answered part (g) by telling me to homogenize without giving any indication that they knew what that meant. Most people, though, reached the last part with little damage done, and many completed it as well. Some people got confused about whether the new homogenising variable should go first or last: it doesn't matter, but once you've made a decision you must stick with it. As homogeneous coordinates are not unique there are many ways of giving the right answer: some people lost marks by mentioning the same point twice under different names. Not very many people got full marks overall but most of you were somewhere near: a good outcome.

Q2. Not everybody remembered the bookwork in section (a) but most did, and just about everybody knows the Nullstellensatz. The bookwork in (c) was a case of all or nothing, in about equal quantities: only one or two people wrote anything that was only partly right. There was some confusion about (d), where some of you wanted to prevent the numerator from being zero too. Part (e) was on an examples sheet and people who attempted it mostly got there. However, 18/20 was a very common score because for some reason that I cannot understand only two people (both of whom had slipped up earlier) saw the extremely easy answer to part (f). You've been given an equation: use it!

Q3. Most of you knew (a) but many of you answered (b) by repeating a phrase from my old notes, which is technically correct but unhelpful in this context. There I was saying what singular means for arbitrary varieties and I talked about the dimension of the tangent space; but you didn't define the tangent space, so you haven't answered the question. Anyway, this question asks about hypersurfaces (all I did this year) and that's much easier. You can't get away with any answer that doesn't mention derivatives. In part (c) some people forgot the characteristic p exception and some forgot how to deal with it, but a commoner mistake was to jump over the main point in characteristic zero, which is the Nullstellensatz. By part (d) the wheels were coming off for a good many of you: you wanted to differentiate the homogeneous polynomial, which is the wrong thing. It was (e) that really settled things, though. The cases are not $z = 1$ and $z = 0$; they are $z = 1$, $y = 1$ and $x = 1$. That's what part (d) tells you. If you look at $z = 0$ you only see the intersection of the curve with a line, i.e. some points: you don't know how they got there so you don't know whether they are singular points of the curve. You have to look *near* the point, and that means affine pieces, not hyperplane sections. Depending which coordinate you chose the wrong method did sometimes give the right answer, but if that happened you didn't get any more marks. By the way, some of you wrote down a polynomial g in x and y (here or in Question 2) and asserted that $g(x, y) = 0$ has no solutions over $\mathbb{C}$: this is strange, as if you choose a value for y (say $y = 1$) then $g(x, 1)$ is a polynomial in x, which you would immediately agree does have zeros in $\mathbb{C}$.

Q4. Only a few answers to this question were handed in and most of them were futile because there were better answers to the other three. There were no good answers at all to the last part, although it is quite easy; but those who could have done it had concentrated their efforts elsewhere. One student, though, saved himself or herself from overall disaster by correctly reproducing the bookwork in part (b)