Summary of MA22017 Groups and Rings

I. Basic theory of groups

I.4 Definition of group (alternative version in I.7); abelian group
I.13 Subgroups, how to recognise them; normal subgroups
I.20 (Group) homomorphism, how to recognise them
I.27 Kernels are normal subgroups
I.28 Injective is the same as trivial kernel
I.29 Isomorphism
I.31 Bijective homomorphisms are isomorphisms
I.36 Order of group, order of element
I.39 Subgroup generated by a set; cyclic groups

II. Structure of groups

II.4 Cosets partition G
II.8 Lagrange's Theorem; also $o(g)$ divides $|G|$
II.16 Quotient group
II.21 Normal subgroups are kernels
II.25 First Isomorphism Theorem; factorisation of homomorphisms

III. Group actions

III.2 Definition of group action; action gives homomorphism $G \to \operatorname{Sym} X$
III.10 Orbits partition X; transitive action
III.14 Stabilisers are subgroups, all subgroups are stabiliser
III.19 Orbit-stabiliser theorem
III.22 Cayley's theorem: all groups are isomorphic to subgroups of permutation groups

IV. Rings

IV.4 Definition of ring; status of 1.
IV.7 Units
IV.12 Integral domains, fields
IV.20 Subrings
IV.25 Ring homomorphisms
IV.29 Evaluation map on polynomials

V. Modules, ideals and quotient rings

V.1 Definition of R-module; linear combinations are what is allowed
V.7 Ideals, principal ideal
V.14 Quotient ring
V.19 Kernels are ideals
V.22 Ideals are kernels
V.23 First Isomorphism Theorem; factorisation of homomorphisms
V.28 Fields do not have interesting ideals so maps out of fields are injective or zero
V.29 Characteristic; for integral domains, it is 0 or a prime
V.36 $I + J$ and IJ; products of rings
V.41 Chinese Remainder Theorem
V.43 Prime ideals, maximal ideals
V.45 R/I is a domain iff I is prime, a field iff I is maximal
V.52 Field of fractions

VI. Factorisation in integral domains

VI.8 Primes, irreducibles, cancellation
VI.10 Primes are irreducible
VI.12 Euclidean domain
VI.16 Euclidean domains are PIDs

VI.19 Minimum polynomials
VI.23 In a PID, irreducibles are prime (and the quotient is a field)
VI.25 UFD
VI.28 In a UFD, irreducibles are prime
VI.29 PIDs are UFDs
VI.33 Counterexamples
VI.36 Content exists
VI.41 product of primitive polynomials is primitive
VI.42 Irreducibles in $R[t]$ and $\mathcal{Q}(R)[t]$
VI.43 $R[t]$ is a UFD if R is a UFD
VI.49 Irreducibility criteria

VII. Structure of modules

VII.3 Noetherian
VII.7 Free modules
VII.21 Torsion
VII.24 Maps with free image split
VII.26 Submodule of free module is free
VII.29 Standard basis for submodule of a free module
VII.30 Classification theorem for finitely generated modules over a PID
VII.33 Decomposition into p parts
VII.36 How to find the modules

VIII. Tensor products

VIII.3 Dual space
VIII.7 Bilinear maps
VIII.14 Tensor product
VIII.23 Tensors
VIII.26 Symmetric and exterior powers
VIII.28 Dimension of symmetric power
VIII.30 Dimension of exterior power
VIII.34 Determinant