GROUPS AND RINGS (MA22017)

SOLUTIONS TO PROBLEM SHEET 0

1. In each of the following cases, either convince yourself that G is a group or give ONE reason why it isn't. (Note: "under $*$" means "taking $*$ as the group operation".)

(a) $G = \mathbb{N}$ under addition.

(b) $G = \mathbb{C}$ under multiplication.

(c) $G = 2\mathbb{Z}$, the set of even integers, under addition.

(d) G is the set of odd integers under addition.

(e) G is the set of odd integers under multiplication.

(f) G is the set of permutations of $\{1, \ldots, 6\}$ that leave 3 fixed.

(g) G is the set of permutations of $\{1, \ldots, 6\}$ that move 3 to 4.

(h) $G = M_{2\times 2}(\mathbb{R})$ under matrix multiplication.

(i) $G = M_{2\times 2}(\mathbb{R})$ under matrix addition.

(j) $\mathbb{R}^3$ under vector cross product.

(k) $G = \mathbb{Z} \setminus \{1, -1\}$ under addition.

Solution:

(a) No inverses.

(b) 0 has no inverse.

(c) Group.

(d) No identity.

(e) No inverse, e.g. 3 *has no inverse.*

(f) Group.

(g) No identity.

(h) No inverses, e.g. the zero matrix has no inverse.

(i) Group.

(j) No identity.

(k) *Not closed.*

2. Read the section on permutations from Algebra 1A again. Compute each of the following products in S_n (for n large enough – say the largest number mentioned), writing the answer as a product of disjoint cycles. Say whether the product is an even permutation or an odd permutation.

(a) (123)(234)

(b) (13)(12)(24)(23)

(c) (12345)(54321)

(d) (1234)(25)(532)

(e) (12)(1234)(12)

Solution:

(a) (12)(34), *even.*

(b) (12)(34), *even – notice that* (13)(12) = (123) *so this is simply (a) again.*

(c) Identity, even.

(d) (12354), *even.*

(e) (2134), *odd – you can multiply it out, or notice that your orders are to swap 1 and 2, do something and swap them back, and that's the same as doing the something with 1 and 2 swapped.*

3. Persuade yourself that S_3 and D_6 are the same group (D_{2n} is the symmetries of an n-gon). How big is this group? Is it the same as $\mathbb{Z}/6$, and why or why not? Is D_8 the same as S_4?

Solution: *Consider the effect on vertices! It has order 6 but it's not abelian so it can't be* $\mathbb{Z}/6$*. And* D_8 *has order 8 but* S_4 *has order 24, so they aren't the same either.*

GKS, 1/2/26