

GROUPS AND RINGS (MA22017)

SEMESTER 2 MATHEMATICS: PROBLEM SHEET 9

*Homework questions, marked **H**, should be handed in according to the directions given by your tutor. Other questions are marked **W** for warmup or work or **E** for extra or enthusiast. A copy of this sheet is on Moodle and at*

<http://people.bath.ac.uk/masgks/MA22017/sheet9.pdf>

1 W Write each of the following $\mathbb{Z}$ -modules in the form $\prod_j \mathbb{Z}/p_j^{k_j} \mathbb{Z}$ with p_j primes.

- (a) $\mathbb{Z}/12\mathbb{Z}$
- (b) $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$
- (c) $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$

2 W Write each of the following $\mathbb{Z}$ -modules in the form $\prod_i \mathbb{Z}/a_i \mathbb{Z}$ with $a_i | a_{i+1}$.

- (a) $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$
- (b) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$
- (c) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/9\mathbb{Z}$

3 H Find all $\mathbb{Z}$ -modules of order $520 = 8 \times 5 \times 13$, expressed as $\prod_i \mathbb{Z}/a_i \mathbb{Z}$ with $a_i | a_{i+1}$. Rewrite each of them in the form $\prod_j \mathbb{Z}/p_j^{k_j} \mathbb{Z}$ with p_j primes.

4 H Find all abelian groups of order 360.

GKS, 15/4/25