

GROUPS AND RINGS (MA22017)

SEMESTER 2 MATHEMATICS: PROBLEM SHEET 7

*Homework questions, marked **H**, should be handed in according to the directions given by your tutor. Other questions are marked **W** for warmup or work or **E** for extra or enthusiast. A copy of this sheet is on Moodle and at*

<http://people.bath.ac.uk/masgks/MA22017/sheet7.pdf>

1 W What is the content of each of the following polynomials?

- (a) $3x^3 - 12x^2 - 9 \in \mathbb{Z}[x]$.
- (b) $3x^3 - 12x^2 - 9 \in \mathbb{Q}[x]$.
- (c) $3wx^3 - 12wx^2 - 9w \in R[x]$, where $R = \mathbb{Z}[w]$.
- (d) $3wx^3 - 12wx^2 - 9w \in S[w]$, where $S = \mathbb{Z}[x]$.
- (e) $3wx^3 + 3x^3 - 12w^2x^2 - 12wx^2 - 9w - 9 \in R[x]$, where $R = \mathbb{Z}[w]$.

2 W Say whether each of the following polynomials is reducible or irreducible, giving reasons.

- (a) $3x^3 - 12x^2 - 9 \in \mathbb{Z}[x]$.
- (b) $3x^3 - 12x^2 - 9 \in \mathbb{Q}[x]$.
- (c) $x^2 + 5x - 3 \in \mathbb{F}_{11}[x]$.
- (d) $x^2 + 5x - 3 \in \mathbb{F}_{13}[x]$.
- (e) $x^2 + 5x - 3 \in \mathbb{F}_{37}[x]$.
- (f) $x^3 + 5x - 3 \in \mathbb{F}_{13}[x]$.
- (g) $x^3 + 5x - 3 \in \mathbb{F}_{11}[x]$.

3 H Say whether each of the following polynomials is reducible or irreducible in $\mathbb{Q}[x]$, giving reasons. You may want to look at Question 2 sometimes.

- (a) $x^4 - 10x^3 - 15 \in \mathbb{Z}[x]$.
- (b) $x^4 - 10x^3 - 15 \in \mathbb{Q}[x]$.
- (c) $x^4 - x^3 - 10x^2 + 7x + 3 \in \mathbb{Q}[x]$.
- (d) $x^4 - 14x^3 + 36x^2 - 34x - 4 \in \mathbb{Q}[x]$.
- (e) $x^3 + 5x - 3 \in \mathbb{Q}[x]$.

GKS, 21/3/25