GROUPS AND RINGS (MA22017)

SEMESTER 2 MATHEMATICS: PROBLEM SHEET 11

Homework questions, marked **H**, *should be handed in according to the directions given by your tutor, which at this stage of the year probably means that they shouldn't be handed in. Other questions are marked* **W** *for warmup or work or* **E** *for extra or enthusiast. A copy of this sheet is on Moodle and at*

`http://people.bath.ac.uk/masgks/MA22017/sheet11.pdf`

1 W Suppose M, N and W are R-modules.

(a) If $\varphi\colon M \to W$ and $\psi\colon N \to W$ are linear maps, is $a\colon M \times N \to W$ given by $a(m,n) = \varphi(m) + \varphi(n)$ a bilinear map?

(b) If W is an R-algebra (see Definition IV.17 and replace K with R) and $\varphi\colon M \to W$ and $\psi\colon N \to W$ are linear maps, is $b\colon M \times N \to W$ given by $a(m,n) = \varphi(m)\varphi(n)$ a bilinear map?

2 H Compute

(a) $\mathbb{Z}/6\mathbb{Z} \otimes \mathbb{Z}/4\mathbb{Z}$

(b) $\mathbb{Z}/2\mathbb{Z} \otimes \mathbb{Z}/3\mathbb{Z} \otimes \mathbb{Z}/4\mathbb{Z}$

(the tensor products are over $\mathbb{Z}$).

3 E If V is an n-dimensional vector space over a field k, the *symmetric algebra* is the ring

$$S^*V = \bigoplus_{d=0}^{\infty} \operatorname{Sym}^d V$$

with the convention that $\operatorname{Sym}^0 V = k$ and the product given by

$$(v_1 \dots v_d)(w_1 \dots w_e) = v_1 \dots v_d w_1 \dots w_e$$

– recall that $v_1 \dots v_d = \sum_{\sigma \in S_d} v_{\sigma(1)} \dots v_{\sigma(d)}$. Show that this makes S^*V into a k-algebra, and that it is isomorphic to $k[x_1, \dots, x_n]$.

GKS, 28/4/25