GROUPS AND RINGS (MA22017)

SEMESTER 2 MATHEMATICS: PROBLEM SHEET 1

Homework questions, marked H, should be handed in according to the directions given by your tutor. A copy of this sheet is on Moodle and at

`http://people.bath.ac.uk/masgks/MA22017/sheet1.pdf`

1 W. In each of the following cases say whether H is a subgroup of G. If it is, say whether it is a normal subgroup.

(a) $G = \mathbb{Z}$ and $H = \{2^k \mid k \in \mathbb{N}\}$

(b) $G = \mathbb{Q}^*$ (with multiplication) and $H = \{2^k \mid k \in \mathbb{Z}\}$.

(c) $G = \mathrm{SL}(2, \mathbb{R})$ and $H = \mathrm{SL}(2, \mathbb{Z})$.

2 H Prove or disprove the following statements.

(a) If G is a group and H and K are subgroups of G, then $H \cap K$ is always a subgroup of G.

(b) If G is a group and H and K are subgroups of G, then $H \cup K$ is always a subgroup of G.

(c) If G is a group and H and K are subgroups of G, and $K \lhd G$ then $H \cap K \lhd G$.

(d) If G is a group and H and K are subgroups of G, and $K \lhd G$ then $H \cap K \lhd H$.

3 W Discuss the question in 1.17: is $\mathbb{Z}/2$ a subgroup of $\mathbb{Z}/6$? Is $\mathbb{Z}/3$ a subgroup of $\mathbb{Z}/6$? Also: is $\mathbb{Z}/2$ a subgroup of $\mathbb{Z}/2 \times \mathbb{Z}/2$?

4 W/H Prove:

(a) **(W)** Lemma 1.27 (the image of a group homomorphism is a group);

(b) **(H)** Lemma 1.29 (a group homomorphism is injective if and only if its kernel is trivial);

(c) **(W)** 1.34 (isomorphism is an equivalence relation on groups: first formulate a precise statement of this).

GKS, 6/2/25