GROUPS AND RINGS (MA22017)

SEMESTER 2 MATHEMATICS: PROBLEM SHEET 0

This sheet is for discussion at the first tutorial. A copy of this sheet is on Moodle and at

`http://people.bath.ac.uk/masgks/MA22017/sheet0.pdf`

1. In each of the following cases, either convince yourself that G is a group or give ONE reason why it isn't. (Note: "under $*$" means "taking $*$ as the group operation".)

(a) $G = \mathbb{N}$ under addition.

(b) $G = \mathbb{C}$ under multiplication.

(c) $G = 2\mathbb{Z}$, the set of even integers, under addition.

(d) G is the set of odd integers under addition.

(e) G is the set of odd integers under multiplication.

(f) G is the set of permutations of $\{1, \ldots, 6\}$ that leave 3 fixed.

(g) G is the set of permutations of $\{1, \ldots, 6\}$ that move 3 to 4.

(h) $G = M_{2\times 2}(\mathbb{R})$ under matrix multiplication.

(i) $G = M_{2\times 2}(\mathbb{R})$ under matrix addition.

(j) $\mathbb{R}^3$ under vector cross product.

(k) $G = \mathbb{Z} \setminus \{1, -1\}$ under addition.

2. Read the section on permutations from Algebra 1A again. Compute each of the following products in S_n (for n large enough – say the largest number mentioned), writing the answer as a product of disjoint cycles. Say whether the product is an even permutation or an odd permutation.

(a) $(123)(234)$

(b) $(13)(12)(24)(23)$

(c) $(12345)(54321)$

(d) $(1234)(25)(532)$

(e) $(12)(1234)(12)$

3. Persuade yourself that S_3 and D_6 are the same group (D_{2n} is the symmetries of an n-gon). How big is this group? Is it the same as $\mathbb{Z}/6$, and why or why not? Is D_8 the same as S_4?

GKS, 1/2/26