ALGEBRA 2B (MA20217)

SEMESTER 3 MATHEMATICS: PROBLEM SHEET 4

Homework questions, marked H, should be handed in by the time agreed with your tutor. A copy of this sheet is on Moodle and at

`http://people.bath.ac.uk/masgks/Algebra2B/sheet4.pdf`

1 W Prove the assertions in III.17(v) in the notes: that in the action of $\mathrm{SL}(2,\mathbb{Z})$ on the upper half-plane $\mathbb{H} = \{z \in \mathbb{C} \mid \operatorname{Im} z > 0\}$, the stabiliser of most $z \in \mathbb{H}$ is $\pm I$, but the stabiliser of $i \in \mathbb{H}$ is a group of order 4 generated by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and the stabiliser of $\omega = e^{2\pi i/3}$ is of order 6, generated by $\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$.

2 H Prove the assertion in the proof of Proposition III.18, that left multiplication by G on $X = \{gH \mid g \in G\}$ defines a group action and that the stabiliser of $1_G H$ is H.

3 W Is it true that if a finite G acts on a set X and the orbits $\mathrm{orb}_G(x)$ and $\mathrm{orb}_G(y)$ are the same size, then $\mathrm{Stab}_G(x) \cong \mathrm{Stab}_G(y)$? Give a proof or a counterexample.

4 H There are fifteen ways of organising six objects into pairs (unordered). Show this directly by counting. Then use the action of the symmetric group S_6 to give a different proof using the orbit-stabiliser theorem.
How many ways are there of organising $2k$ objects into k pairs?

GKS, 8/3/24