# ALGEBRA 2B (MA20217)

## SEMESTER 2 MATHEMATICS: PROBLEM SHEET 2

*Homework questions, marked H, should be handed in by the deadline set by your tutor. A copy of this sheet is on Moodle and at*

`http://people.bath.ac.uk/masgks/Algebra2B/sheet2.pdf`

**1 W** In each of the following cases say what the order $|G|$ of $G$ is.

(a) $G = S_5$, the symmetric group on 5 letters.

(b) The alternating group $A_5$.

(c) $\mathbb{Z}/n$.

(d) The subgroup $n\mathbb{Z}$ of $\mathbb{Z}$

**2 H** In each of the following cases say what the order $o(g)$ of $g$ is. Verify that $o(g)$ divides $|G|$.

(a) $G = S_5$ and $g = (13)(245)$.

(b) $G = \mathrm{SL}(2, \mathbb{F}_2)$ and $g = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. (By $\mathbb{F}_2$ we mean $\mathbb{Z}/2$ considered as a field, i.e. with both addition and multiplication mod 2.)

**3 W** Suppose that $G$ is a cyclic group of finite order $n$ and that $g$ generates $G$ (in this case, we say that $g$ is a generator). Is $g^2$ a generator? Which other elements of $G$ are generators? How many of them are there?

**4 W** By considering left and right cosets, show that if $H < G$ and $|G : H| = 2$ then $H \lhd G$. Give an example of a group $G$ and a non-normal subgroup of index 3.

**5 H** Suppose that $G$ is a group and $H$ is a subgroup.

(a) Verify that there is no such thing as "left index" and "right index": the number of left cosets of $H$ in $G$ is equal to the number of right cosets. You may wish to consider the map $gH \mapsto Hg^{-1}$.

(b) Suppose that every non-identity element of $G$ has order 2. Show that $G$ is abelian.

(c) Show that the alternating group $A_4$, which is of order 12, does not have a subgroup of order 6: hence the converse of Lagrange's Theorem is false in general.

GKS, 26/2/24