FURTHER MATHEMATICAL TECHNIQUES (MA10193)
EXAMPLES SHEET 2

I, or somebody, will look at work given to me at the end of the examples class at 12:15 on Wednesday, 9th March, or left before then in the folder on my office door (1W3.35). If you do not have a copy of this sheet, you can find one at
`http://www.bath.ac.uk/~masgks/MA10193/sheet1.ps` *(or* `.dvi` *or* `.pdf`*).*

1. Solve the following set of simultaneous equations using elementary row operations:

$$\begin{aligned} x + 2y - 8z &= 0 \\ 2x - 3y + 5z &= 0 \\ 3x + 2y - 12z &= 0 \end{aligned}$$

2. Use the method of Gauss-Jordan elimination to solve the following set of simultaneous equations:

$$\begin{aligned} 9x + 4y + 3z &= -1 \\ 5x + y + 2z &= 1 \\ 7x + 3y + 4z &= 1 \end{aligned}$$

3. Use elementary row transformations to find the solution to the following simultaneous equations in the case where $a_{11}a_{22} - a_{12}a_{21} \neq 0$.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

4. Find the rank of A where

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 0 & 3 \\ 2 & -4 & 2 \end{pmatrix}.$$

Hence (without solving the system) state how many solutions, if any, the following linear system has:

$$\begin{aligned} 2y - z &= 1 \\ x + 3z &= 3 \\ 2x - 4y + 2z &= -7 \end{aligned}$$

5. Solve the equation (remember that $\det A$ or $|A|$ means the determinant of A)

$$\begin{vmatrix} x+1 & x & x-4 \\ 2 & 1 & -4 \\ 3 & 5 & 1 \end{vmatrix} = 0$$

6. Consider the matrix

$$A = \begin{pmatrix} -1 & 3 & x \\ -2 & 2 & 1 \\ x & 4 & 0 \end{pmatrix}.$$

Calculate $\det A$ in terms of x and hence determine those values of x (to 2 d.p.) for which the matrix has rank less than 3. If $x = 2$ write down the rank of A.

GKS, 2/03/05