MATHEMATICS 2 (MA10193)
EXAMPLES SHEET 1

I, or somebody, will look at work given to me at the end of the examples class at 12:15 on Wednesday, 2nd March, or left before then in the folder on my office door (1W3.35). If you do not have a copy of this sheet, you can find one at
`http://www.bath.ac.uk/~masgks/MA10193/sheet1.ps` *(or* `.dvi` *or* `.pdf`*).*

1. Suppose that the matrix

$$A = \begin{pmatrix} \cos w & p \\ \sin w & q \end{pmatrix}$$

satisfies the equation $AA^T = I$. Find the values of p and q.

2. Check, by calculation, that $(AB)^T = B^T A^T$, where

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 3 & 2 & 1 & -2 \\ 0 & 2 & 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 3 \\ -1 & 2 \\ 3 & 0 \\ 1 & 1 \end{pmatrix}.$$

3. Expand $(A+B)^2$. Check that your expansion is correct in the case where

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}.$$

4. Suppose that

$$A = \begin{pmatrix} 1 & x & 1 \\ x & 2 & y \\ 1 & y & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & -3 & z \\ -3 & 2 & -3 \\ z & -3 & 1 \end{pmatrix},$$

and AB is symmetric. Is it true that $AB = BA$ in this case? Find x, y and z (this is a bit harder).

5. Let

$$B = \begin{pmatrix} 4 & 1 & -2 \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{pmatrix}.$$

Represent B as the sum of a symmetric matrix and a skew-symmetric matrix.

6. Let

$$A = \begin{pmatrix} 2 & 5 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 \\ -3 & 0 \\ 1 & 4 \\ 3 & 2 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$$

Say which of the following expressions are defined, and calculate those that are:

a) A^2, B^2, C^2.

b) AA^T, B^TB, CC^T, C^TC.

c) C^TAB, $(AB)^TC$.

7. Suppose that

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \text{ and } B = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}.$$

a) Verify that $AB = BA = I$.

b) Use part (a) and the rules about multiplication and transposes of matrices to write down the value of A^TB^T.

c) Use (a) and (b) to write each of the following systems of equations in matrix form and thereby solve them:

$$\begin{aligned} -11x + 2y + 2z &= 1 \\ -4x + z &= 2 \\ 6x - y - z &= 3 \end{aligned}$$

and

$$\begin{aligned} -11x - 4y + 6z &= 1 \\ 2x - z &= 2 \\ 2x + y - z &= 3. \end{aligned}$$

GKS, 22/02/05