MATHEMATICS 2 (MA10193)
EXAMPLES SHEET 3: SOLUTIONS

1. $(7-2i)+(8+5i)=15+3i$; $(3-i)(6+5i)=23+9i$; $(3-i)/(6+5i)=\frac{13-21i}{65}$; $(2+i)/i=1-2i$; $\sqrt{-i}=\pm\frac{1+i}{\sqrt{2}}$.

2.
$$A^{-1}=\frac{1}{3}\begin{pmatrix}1 & -1 & 1\\ 5- & -2 & -1\\ -7 & 4 & 2\end{pmatrix}.$$

3. $z^2=8+6i$. You should be able to draw the points (3,1) and (8,6) without help.

4. $z_1z_2=(1-2i)(4+yi)=4+yi-8i+2y$, so the real part is $4+2y$ which is zero if and only if $y=-2$.
$z_1/z_2=(1-2i)/(4+yi)=(1-2i)(4-yi)/(16+y^2)$. Since $16+y^2$ is real, this is real if and only if the numerator, $(1-2i)(4-yi)$, is real; that is, if its imaginary part is zero. But the imaginary part of $(1-2i)(4-yi)=4+2y-yi-8i$ is $-y-8$, so we need $y=8$.

5. $|z|=\sqrt{\sqrt{3}^2+1^2}=2$, and $\tan\arg z=1/\sqrt{3}$ so $\arg z=\pi/3$ (since z is in the first quadrant). Note: it is $\pi/3$ exactly and a calculator approximation gives the wrong answer. The smallest positive integer n such that z^n is a real number is 3, since $\arg z^3=3\arg z=\pi$ (this is why an approximation is wrong: we need to get exactly π here, not something close to it).

6. If $x^2-5x+32=0$ then $x=\frac{1}{2}(5\pm i\sqrt{103})$. The eigenvalues of $\begin{pmatrix}3 & 13\\ -2 & 2\end{pmatrix}$ are found by solving the equation $\begin{vmatrix}3-x & 13\\ -2 & 2-x\end{vmatrix}=0$ and that is $x^2-5x+32=0$ so those are the eigenvalues.

GKS, 9/05/05