

Preface

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This book is a gentle and relaxed introduction to the two branches of pure mathematics which dominate the early stages of the subject as it is taught to undergraduates in many countries. It is not a substitute for more advanced texts, and has no pretensions to comprehensiveness. There are several places where I would have liked to press on further, but you have to stop somewhere. It should, I hope, be easy to read, and to this end the style is decidedly more colloquial than is traditional in text books. I only hope that the language will not date too quickly. Thus this is not a book in the relentless theorem–proof style; it contains discursive commentary.

The ways in which pure mathematicians express themselves, and the cumulative nature of the subject, may make pure mathematics seem daunting to a beginner. The mathematical mode of expression and the deductive method are vital to pure mathematics. We wish to explore strange geometries, new algebraic systems, and infinite dimensional spaces. There is no point in embarking on this enterprise unless we are prepared to be ruthlessly precise, since otherwise, no-one will have any idea what we are talking about (if anything).

These exotic spaces and objects are not part of everyday experience, unlike, for example a dog. If we mention that “there is a dog in the garden”, we do not expect the response “what is a *dog*, what is a *garden*, what does *is* mean in this sentence, why have you used the indefinite article, and what is the contribution of the word *there*?” We know a lot about dogs and gardens, and do not need to put the sentence under scrutiny in order to understand the meaning. However, if instead someone says “every linear group is either virtually solvable, or contains a free subgroup of rank 2”, then either you have to live in a world where these terms are as familiar as dogs and gardens, or you have to take the remark apart, and analyze every part of it until you understand what it asserts. Of course there is little point in doing this unless you happen to know that linear groups are very interesting – which, incidentally, they are.

There is a web site which supports this book.

<http://www.maths.bath.ac.uk/~masgcs/book1/>

If that ever changes, a link to the new site will be put in place. At the web site

you will find additional exercises and solutions, and corrections to any errors that are discovered after publication.

The material in this book is not always arranged in a logically perfect sequence. This is deliberate, and is a consequence of trying to make the book accessible. The ideal way to read the book is from cover to cover. Chapter 1 establishes notation and sets the scene, and Chapter 2 concerns mathematical proof – many readers will want to read that before proceeding with the main topics. I have tried to make subsequent chapters (fairly) independent, though Chapter 6 should definitely be read before either Chapter 7 or Chapter 8. In consequence of the partial independence, some material is repeated in different parts of the book, though the treatments vary.

I also felt that this book should contain early chapters on complex numbers and matrices. These topics are basic to university mathematics, engineering and science, and are rarely or barely taught at secondary level.

It is standard practice to thank everyone who had anything to do with the creation of a book. I therefore wish to thank and congratulate my parents Eileen and Roy. This volume is being published in the year of their golden wedding anniversary, and Springer-Verlag have kindly agreed to celebrate this event by supplying a cover of appropriate colour.

Inductively, if I thank anyone, I thank their parents too, thereby acknowledging the assistance of a vast number of people, creatures, single-celled organisms and amino acids. Please note that this (thinly spread) gratitude was expressed with considerable economy (see Section 2.1).

Despite the millions of generations who have already been thanked, there are some deserving cases who have not yet qualified. I also acknowledge the help and support of various colleagues at the University of Bath. My \TeX and \LaTeX guru is Fran Burstall, and Teck Sin How provided figures at amazing speed. James Davenport and Aaron Wilson helped to weed out errors, and suggested improvements. I must also thank John Toland who persuaded me to write this book with his usual combination of charm, flattery and threats – and supplied the beautiful question that constitutes Exercise 8.4. Any remaining errors are mine, and copyright.

I would also like to thank my bright, enthusiastic and frustrated students, without whom this book would not have been necessary, and my wife Olga Markovna Tabachnikova, without whom my salary would have been sufficient.

GCS, Bath, 11-xi-1997.