1. Consider the series $\sum_{i=1}^{\infty} a_i(-1)^{i+1}$ where for all i we have $a_i \geq 0$ and the sequence (a_i) converges to 0. Prove that the series is convergent. *Hint: Let* $s_m = \sum_{i=0}^{m} a_i$. *Prove that* $(s_{2n}$ *is monotone increasing and bounded above by* a_1. *Do something similar with* $(s_{2n-1}$. *Then apply a theorem to each of these two sequences, and then stitch the results together somehow.*

2. Consider the sequence (t_i) where

$$t_m = \sum_{i=1}^{m} 1/i - \int_1^m x^{-1} dx.$$

Show that $t_n > 1/n > 0 \forall n \in \mathbb{N}$, and moreover that $t_n > t_{n+1} \forall n \in \mathbb{N}$. Conclude that the sequence whose n-th term is

$$1 + 1/2 + 1/3 + \ldots + 1/n - \log n$$

is convergent. (The limit is often called γ, or *Euler's number* or *Euler's constant*).