

Introductory Mathematics: Algebra and Analysis

G. C. Smith

February 5, 1998

Preface	v
1. Sets, Functions and Relations	1
1.1 Sets	1
1.2 Subsets	2
1.3 Well-known Sets	3
1.4 Rationals, Reals and Pictures	6
1.5 Set Operations	8
1.6 Sets of Sets	11
1.7 Paradox	14
1.8 Set-theoretic Constructions	15
1.9 Notation	16
1.10 Venn Diagrams	17
1.11 Quantifiers and Negation	19
1.12 Informal Description of Maps	21
1.13 Injective, Surjective and Bijective Maps	22
1.14 Composition of Maps	23
1.15 Graphs and Respectability Reclaimed	29
1.16 Characterizing Bijections	30
1.17 Sets of Maps	31
1.18 Relations	31
1.19 Intervals	37
2. Proof	39
2.1 Induction	39
2.2 Complete Induction	43
2.3 Counter-examples and Contradictions	47
2.4 Method of Descent	50
2.5 Style	53
2.6 Implication	54
2.7 Double Implication	54
2.8 The Master Plan	56

3. Complex Numbers and Related Functions	57
3.1 Motivation	57
3.2 Creating the Complex Numbers	62
3.3 A Geometric Interpretation	70
3.4 Sine, Cosine and Polar Form	76
3.5 e	80
3.6 Hyperbolic Sine and Hyperbolic Cosine	85
3.7 Integration Tricks	88
3.8 Extracting Roots and Raising to Powers	89
3.9 Logarithm	90
3.10 Power Series	92
4. Vectors and Matrices	95
4.1 Row Vectors	95
4.2 Higher Dimensions	97
4.3 Vector Laws	98
4.4 Lengths and Angles	98
4.5 Position Vectors	103
4.6 Matrix Operations	104
4.7 Laws of Matrix Algebra	106
4.8 Identity Matrices and Inverses	108
4.9 Determinants	110
4.10 Geometry of Determinants	119
4.11 Linear Independence	120
4.12 Vector Spaces	121
4.13 Transposition	123
5. Group Theory	125
5.1 Permutations	125
5.2 Inverse Permutations	130
5.3 The Algebra of Permutations	131
5.4 The Order of a Permutation	133
5.5 Permutation Groups	135
5.6 Abstract Groups	136
5.7 Subgroups	142
5.8 Cosets	143
5.9 Cyclic Groups	146
5.10 Isomorphism	148
5.11 Homomorphism	151

6. Sequences and Series	153
6.1 Denary and Decimal Sequences	153
6.2 The Real Numbers	156
6.3 Notation for Sequences	158
6.4 Limits of Sequences	160
6.5 The Completeness Axiom	169
6.6 Limits of Sequences Revisited	170
6.7 Series	174
7. Mathematical Analysis	181
7.1 Continuity	181
7.2 Limits	194
8. Creating the Real Numbers	197
8.1 Dedekind's Construction	197
8.2 Construction via Cauchy Sequences	199
8.3 A Sting in the Tail: p -adic numbers	202
Further Reading	204
Solutions	205
Index	213