RI Masterclass 11th March 2023

Permutations Workshop

Permutations of 1, 2, 3

There are 6 permutations of the numbers 1, 2, 3, listed below.

$\iota = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$\sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ This is the same as σ^2.

$\tau = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$\tau_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ This is the same as $\sigma\tau$.

$\tau_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ This is the same as $\sigma^2\tau$.

WORKED EXAMPLE

Suppose we want to work out what $\sigma\tau$ looks like. This means we are calculating

$$\sigma\tau = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

To do this, we work **from right to left**. First we see what happens to the number 1.

First, the permutation τ sends 1 to 2. Then the permutation σ sends 2 to 3. Overall $\sigma\tau$ sends 1 to 3.

Next, the permutation τ sends 2 to 1. Then the permutation σ sends 1 to 2. Overall $\sigma\tau$ sends 2 to 2. It stays fixed.

Finally, the permutation τ sends 3 to 3. Then the permutation σ sends 3 to 1. Overall $\sigma\tau$ sends 3 to 1. So we can see that $\sigma\tau$ is the same thing as τ_2!

TASK. Fill out the 'multiplication' table on your worksheets for the permutations listed above. Write all entries as one of the following forms,

ι if the permutation is ι,
σ^m for m=1 or m=2 if the permutation is one of σ or σ^2,
τ if the permutation is τ,

Write any other entry as $\sigma^m\tau^n$ in **this order**. For example **don't** write $\tau\sigma$, you will be able to write this as $\sigma^m\tau^n$ instead but you will need to find m and n.

A good place to start is to work out what $\tau\sigma$ is! Remember we work right to left, so $\tau\sigma$ means to do σ first, then τ.

	σ
τ	$\sigma\tau$

When filling an entry in your table, it should look like this. If τ is on the side row and σ is on the top row, the entry will be $\sigma\tau$.

We worked out that this was the same as τ_2, but we prefer to denote τ_2 as $\sigma\tau$ as described above.

CAREFUL! This means 'do τ first, then do σ'.