

# Why do things sometimes change so fast?

## Tipping Points in Life and Nature

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**BATH**

**Do you want to live in interesting times?**



寧為太平犬，不做亂世人

pinyin: níng wéi tài píng quǎn, bù zuò luàn shì rén

*"It's better to be a dog in a peaceful time than be a man in a chaotic period"*

Sometimes things change slowly

Boring but safe

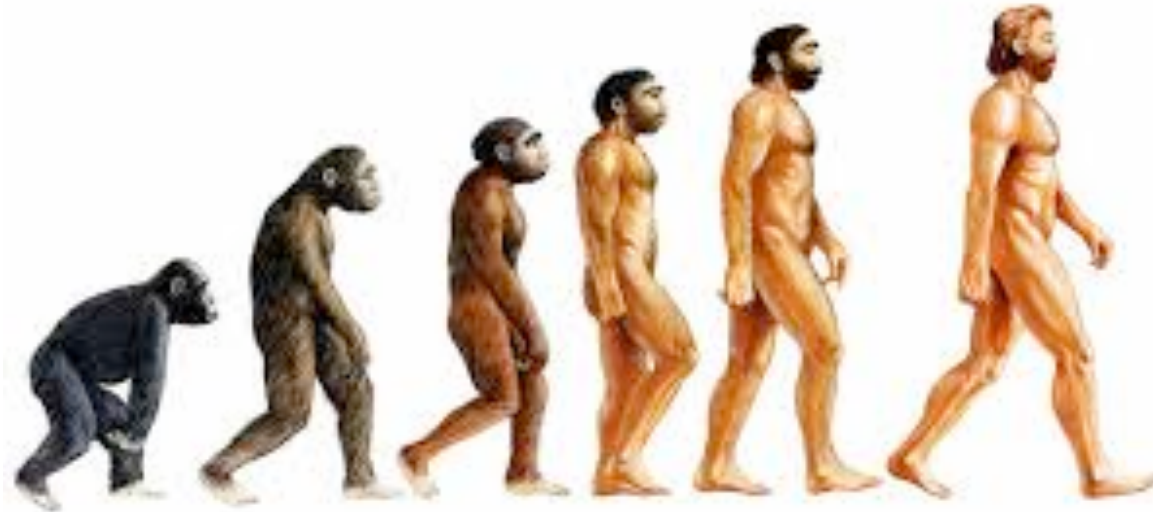


Sometimes they change quickly

Interesting but dangerous



Darwin's theory of evolution seemed to imply that things happen gradually



Lots of other things are like this ..

Even the weather 70% of the time!



WINTER IN THE HIGHLANDS OF SCOTLAND.



SUMMER IN THE HIGHLANDS OF SCOTLAND.

But sometimes things (including the weather) can change very quickly ..



Why do we see sometimes see sudden changes?



# Tipping points: where change happens

A System is in a state of balance

Small change to it leads to an irreversible large change

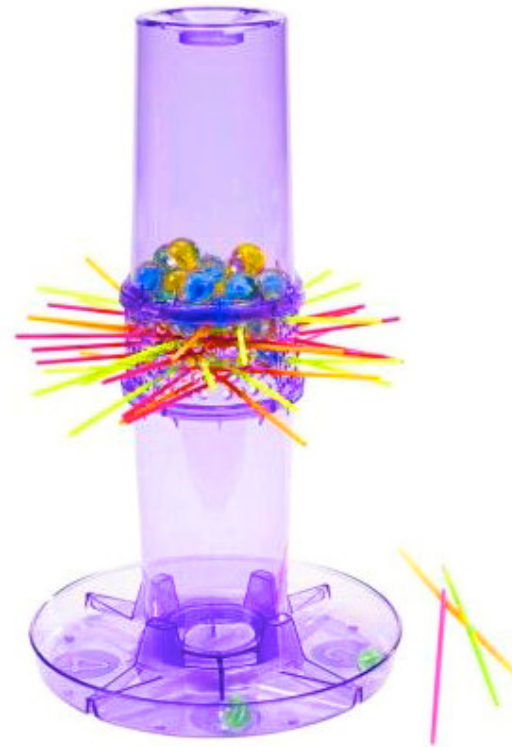






# Some examples of tripping points in life and nature

Kerplunk



Trigger: The final straw

# Avalanches and Sandpiles

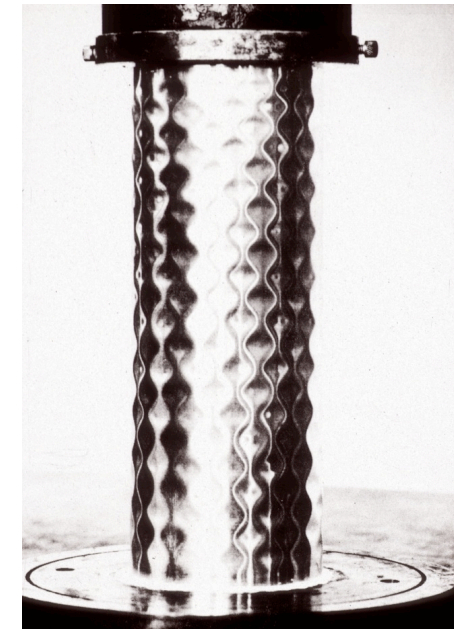
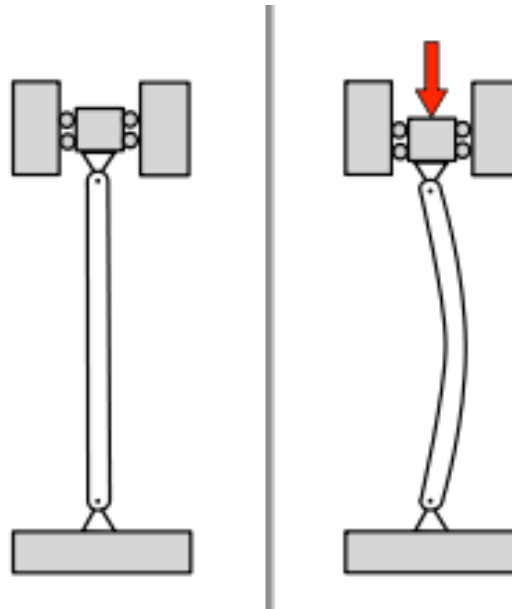


Trigger ... small change in temperature or pressure

# Buckling eg. Coke can

Tipping point:

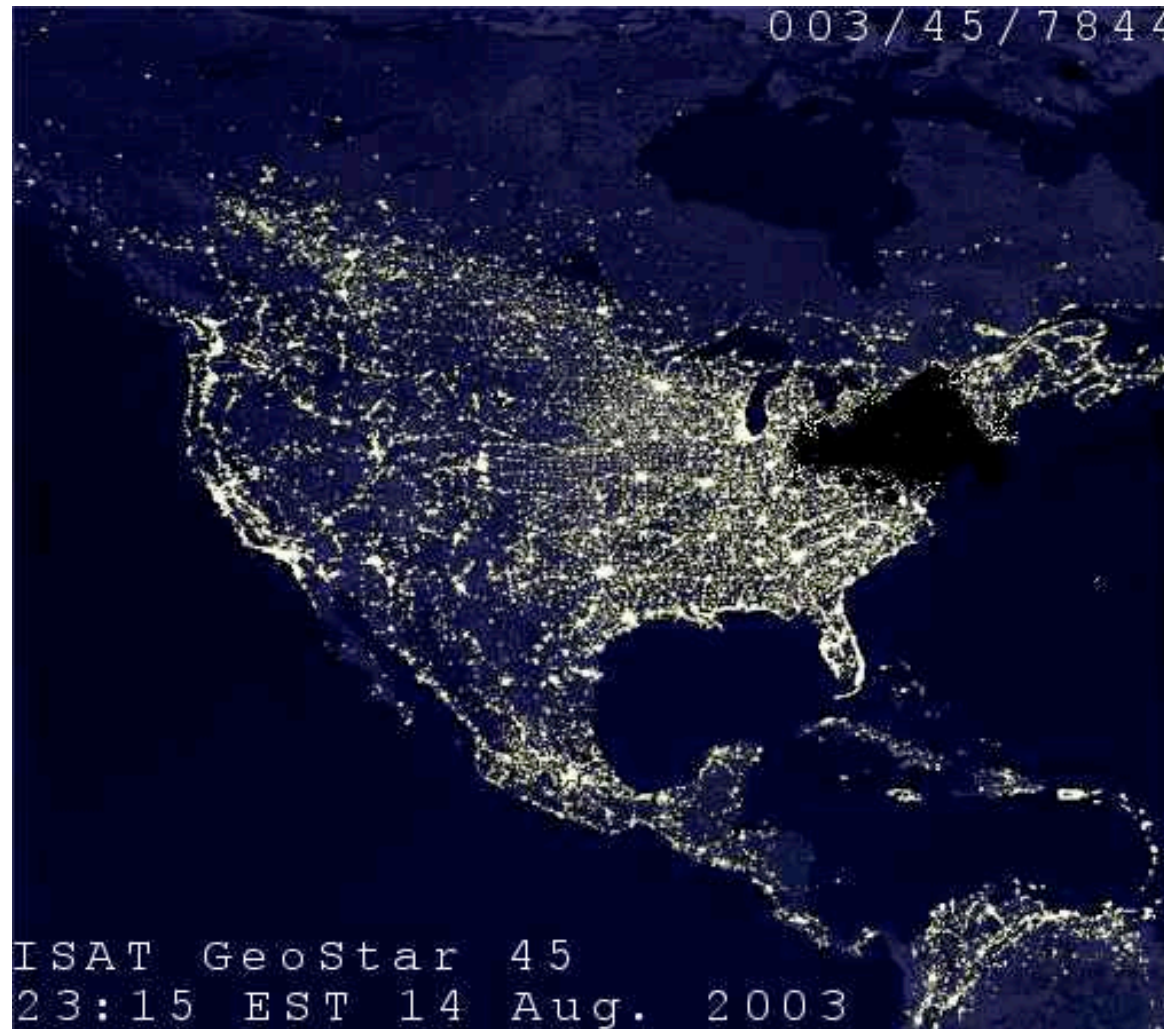
Just too much force, heat or an imperfection



# Rock folding

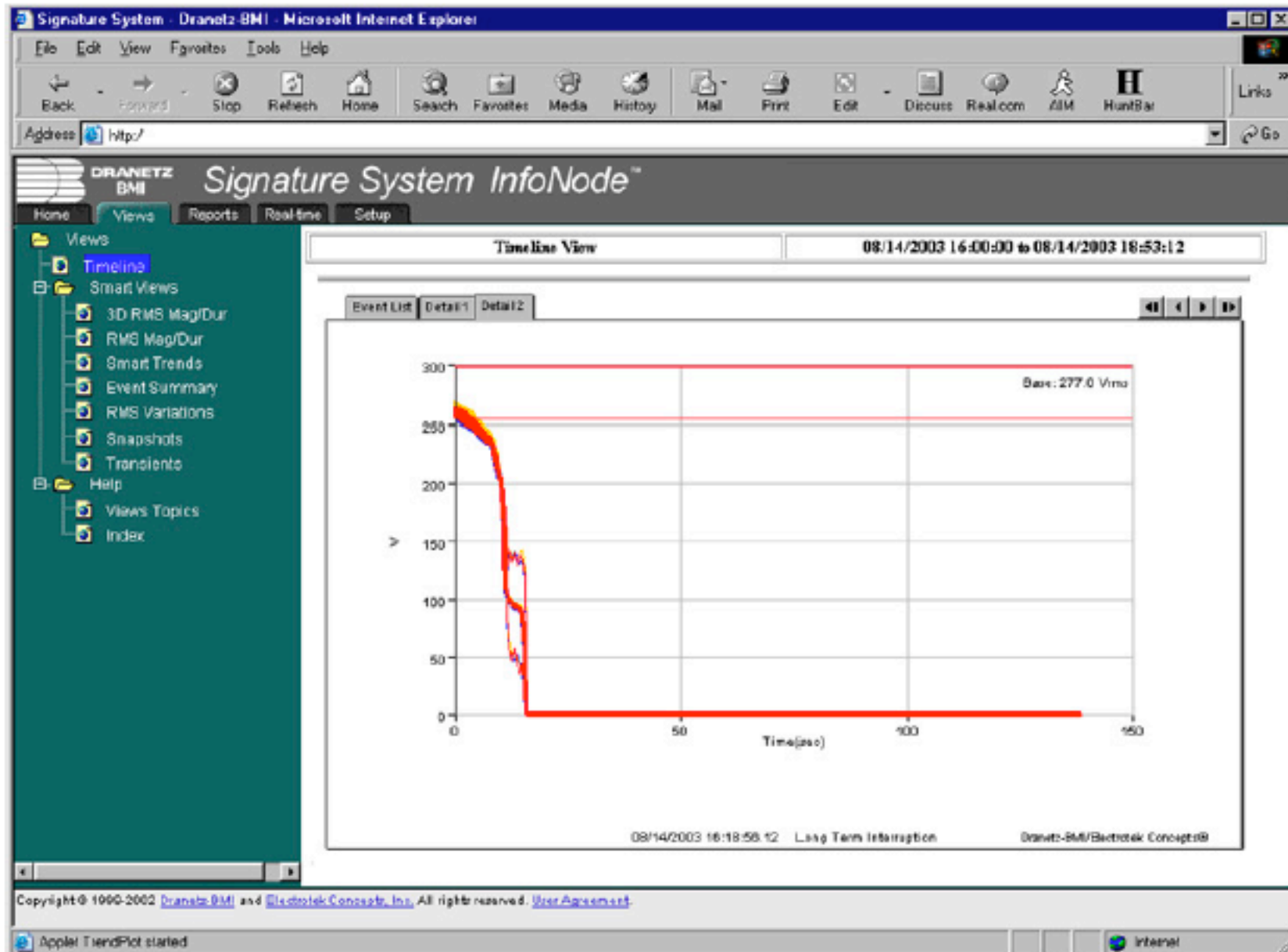


# New York Blackout 2003

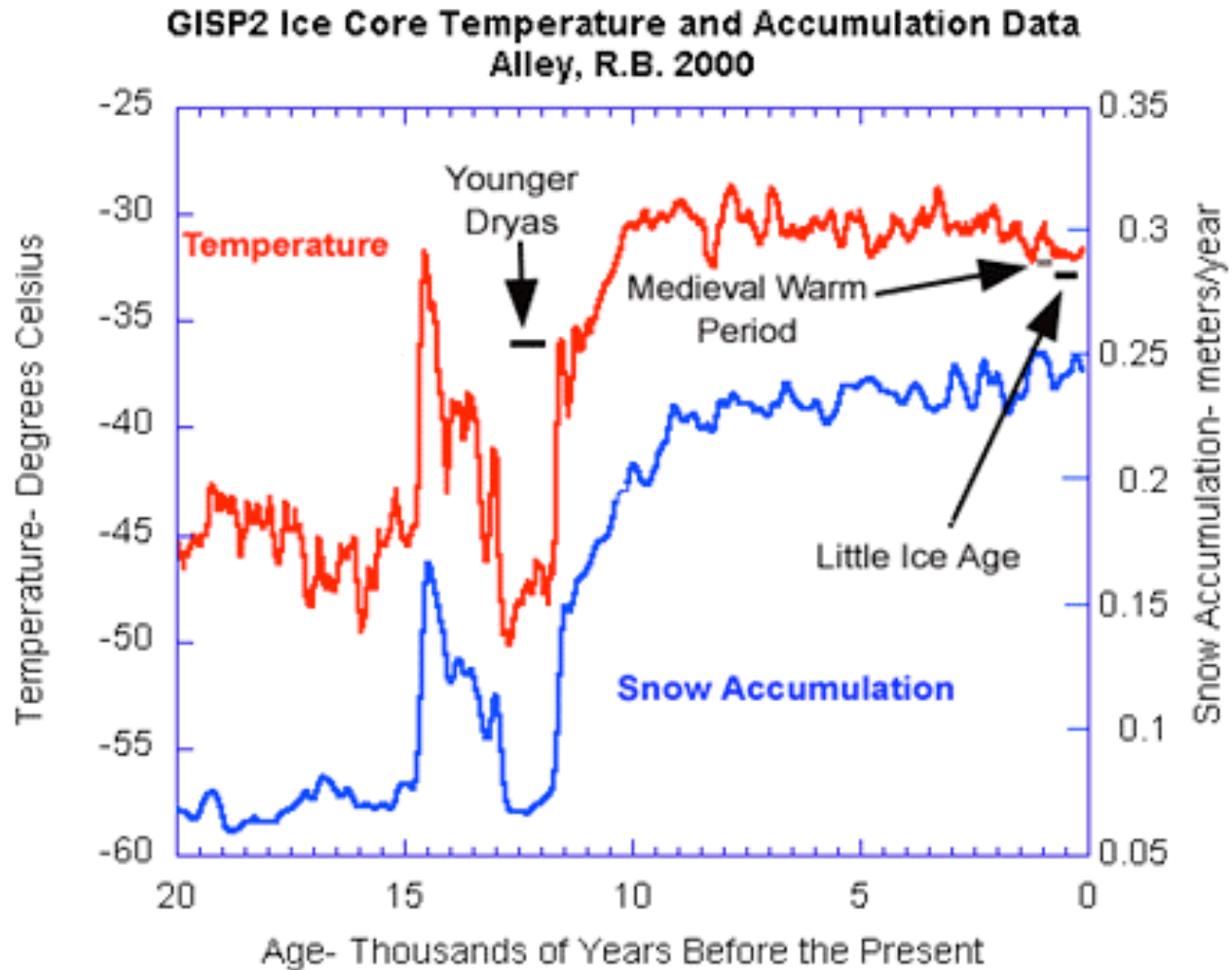


Tipping point: series of local line failures .. Global cascade

# Voltage Collapse at a Site on Staten Island



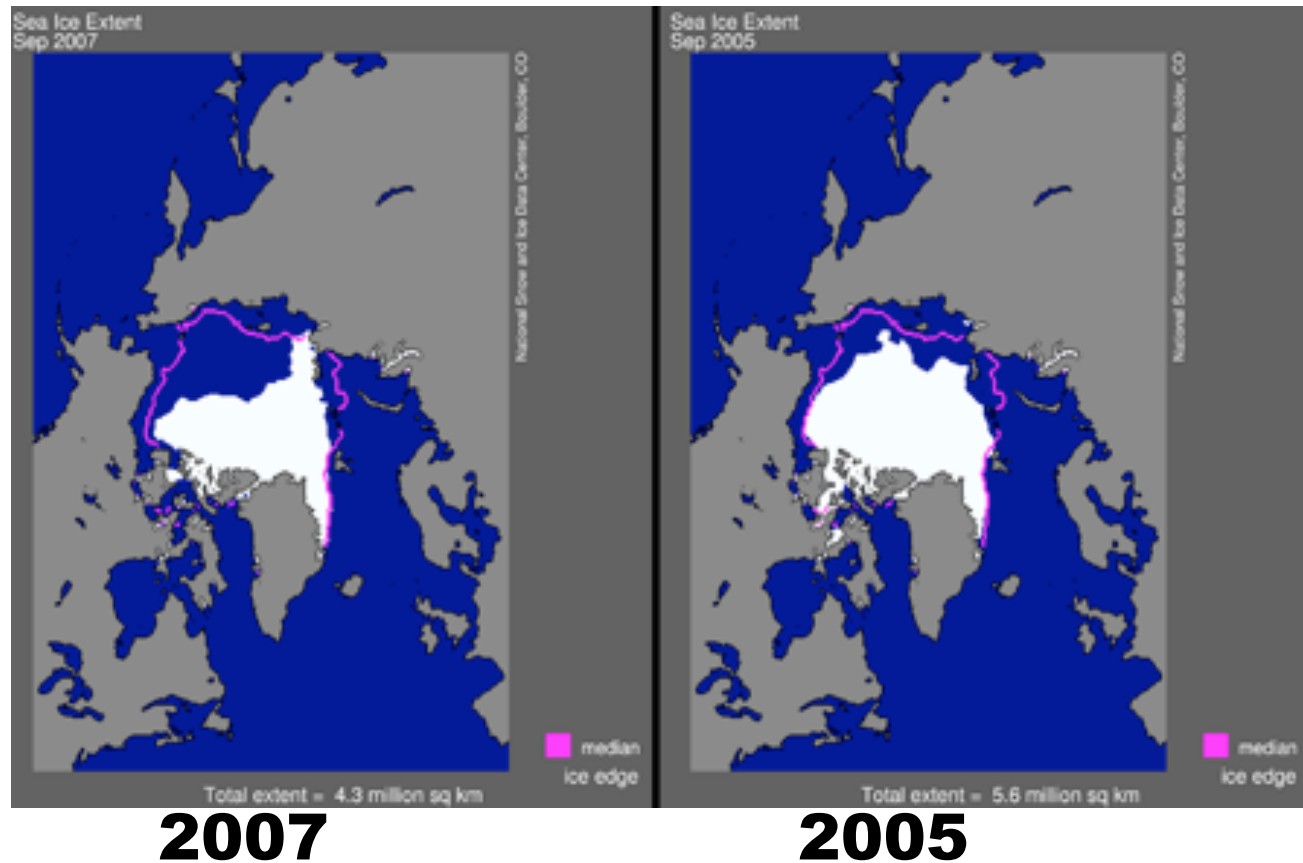
## Paleoclimate: Younger Dryas Ice Age



Tipping point: ?? Shut down in the Atlantic conveyor



## Current climate: The Melting Arctic ?



Every year we lose Arctic ice the size of Scotland!

Tipping point: Too much man made CO<sub>2</sub>??

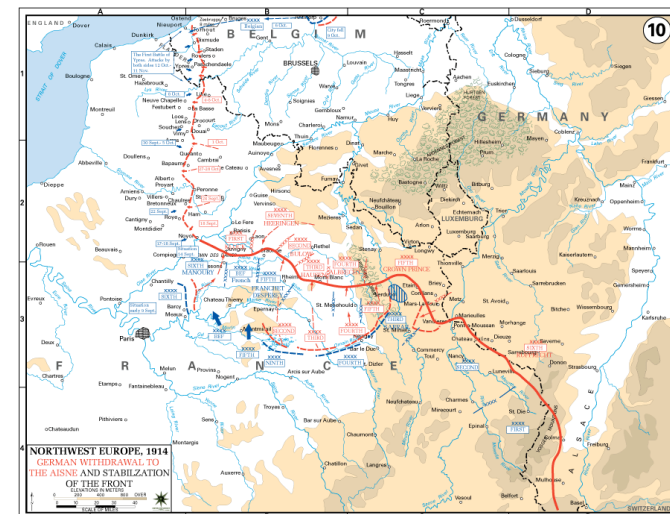
# World War One

Failure of deterrence

Tipping point:

Shooting of Franz Ferdinand at Sarajevo  
by Gavrilo Princep

German Railway Timetables



# A theory for tipping points

Apple cart has two **stable states** separated by an **unstable state**

A ball in a hole is in a **stable state**

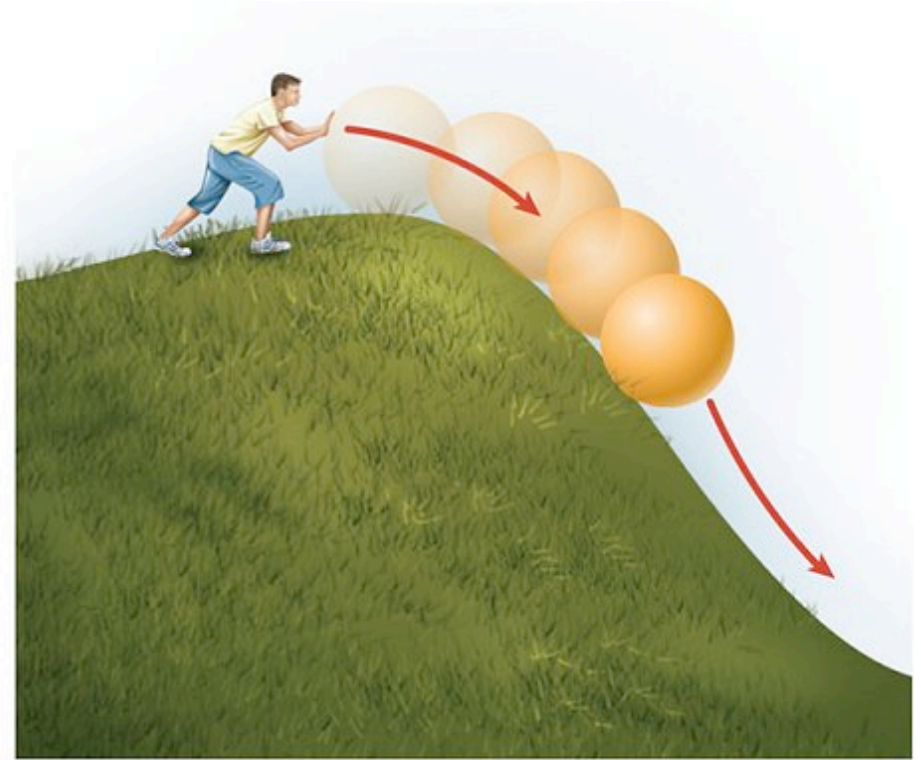


Small change leads to small effect

# A ball on the top of a hill is unstable



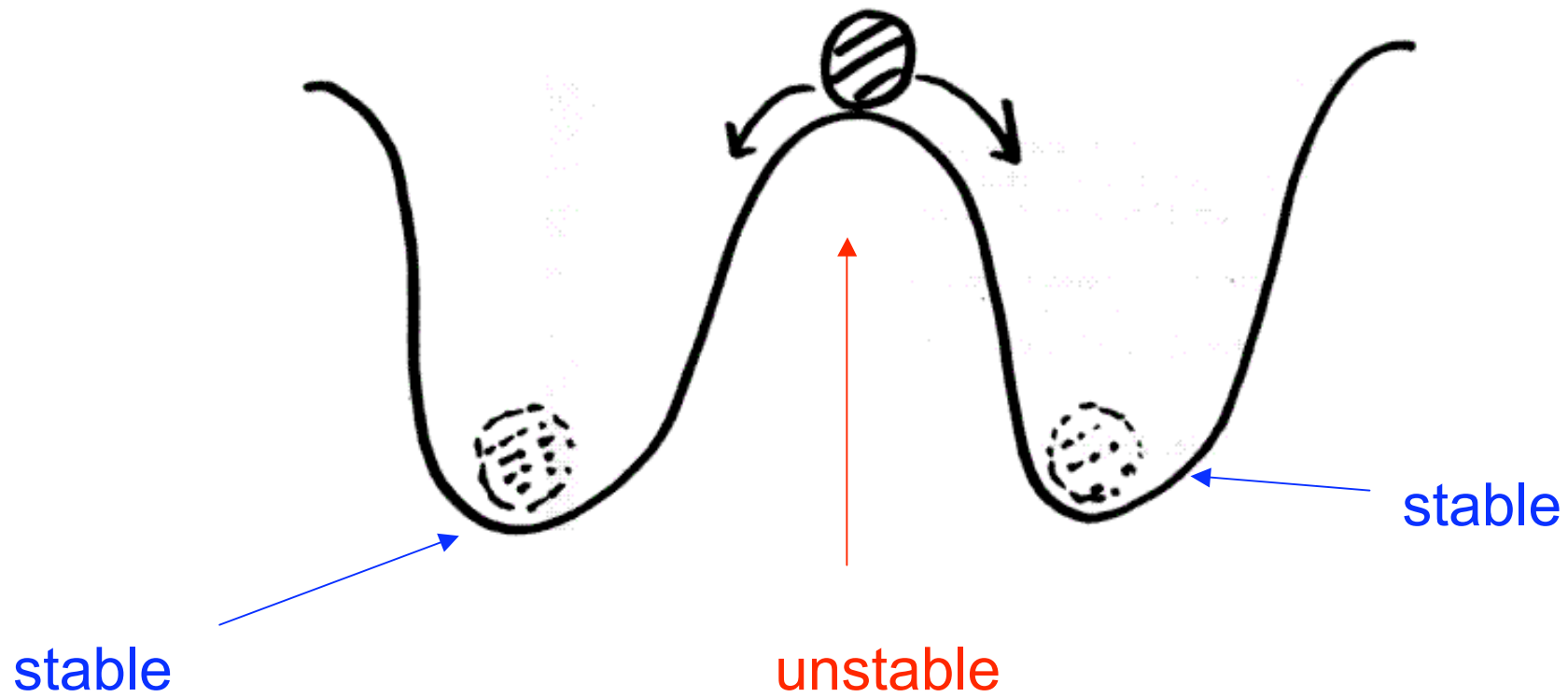
a) Potential energy



(b) Kinetic energy

Small change leads to large effect

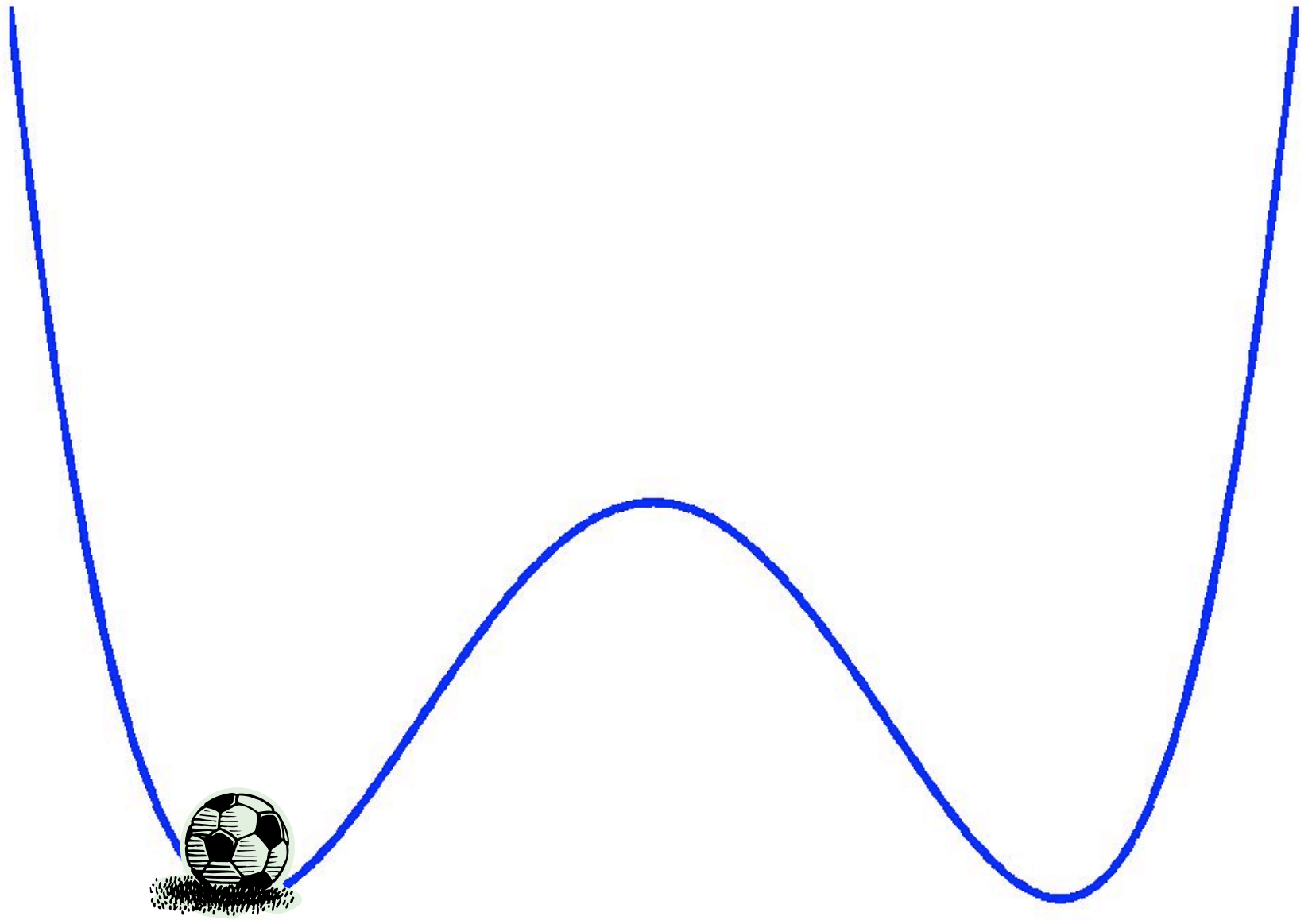
The apple cart is like a ball on a hill between two holes

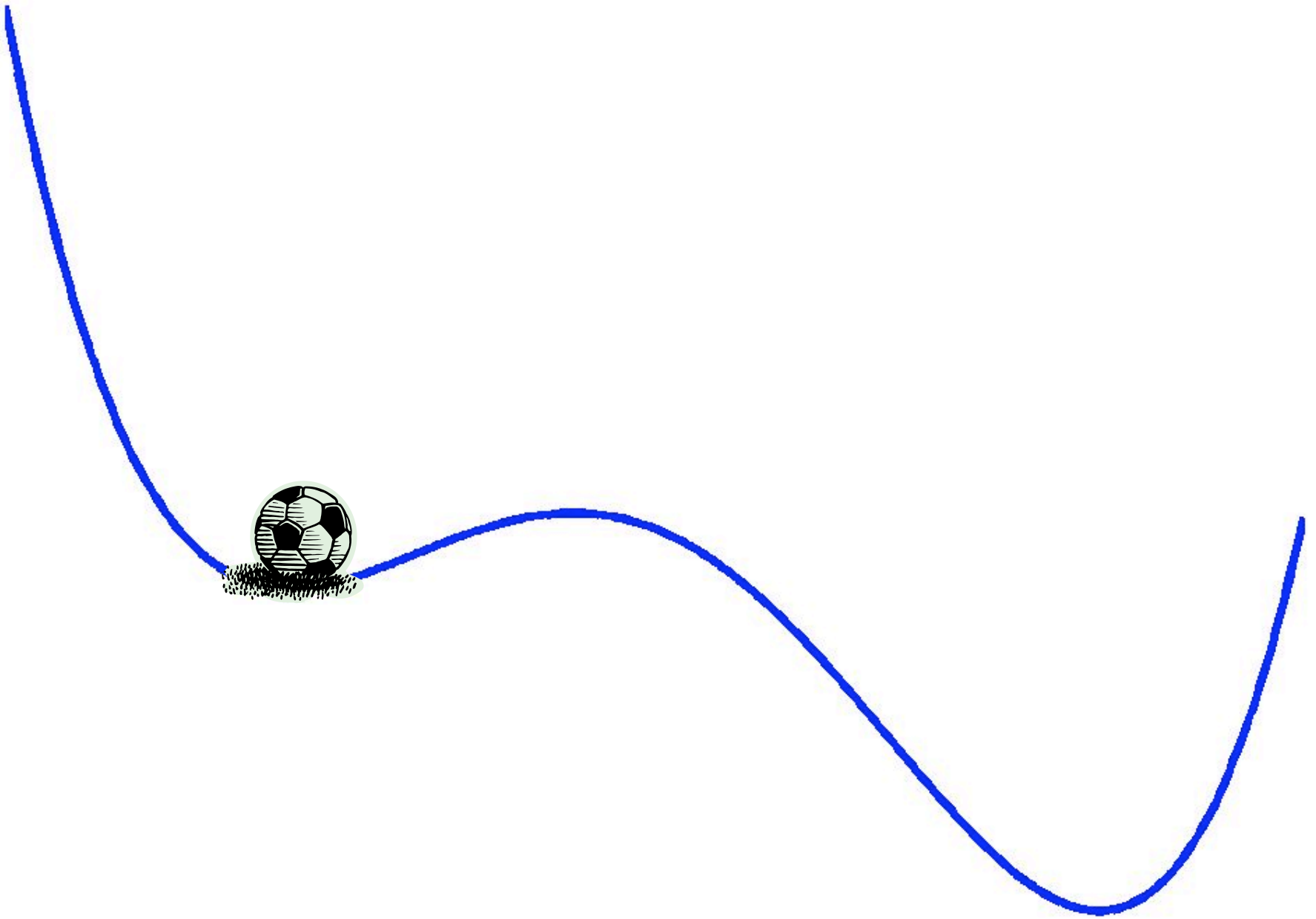


But .. If we **change the shape of the mountain**

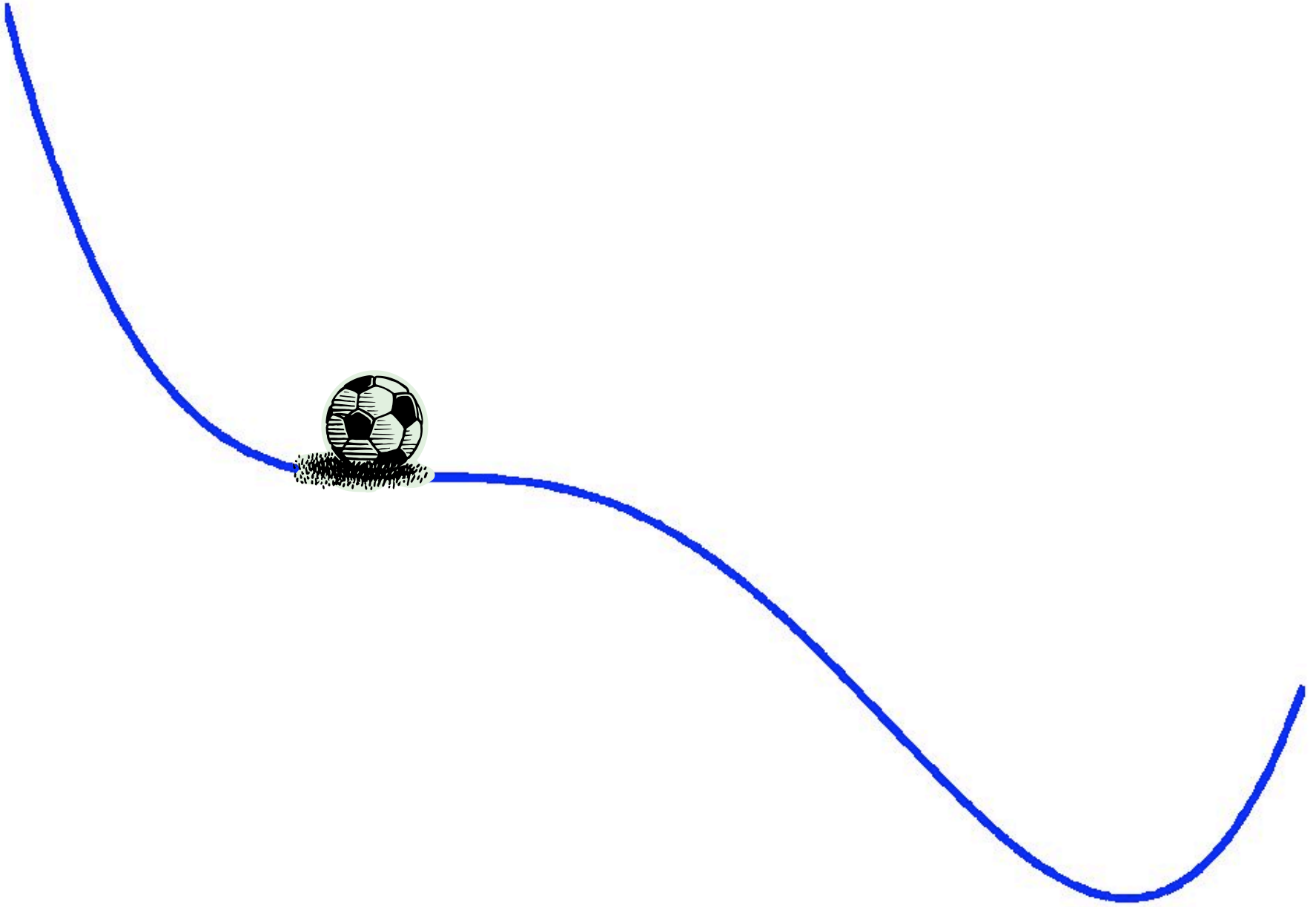


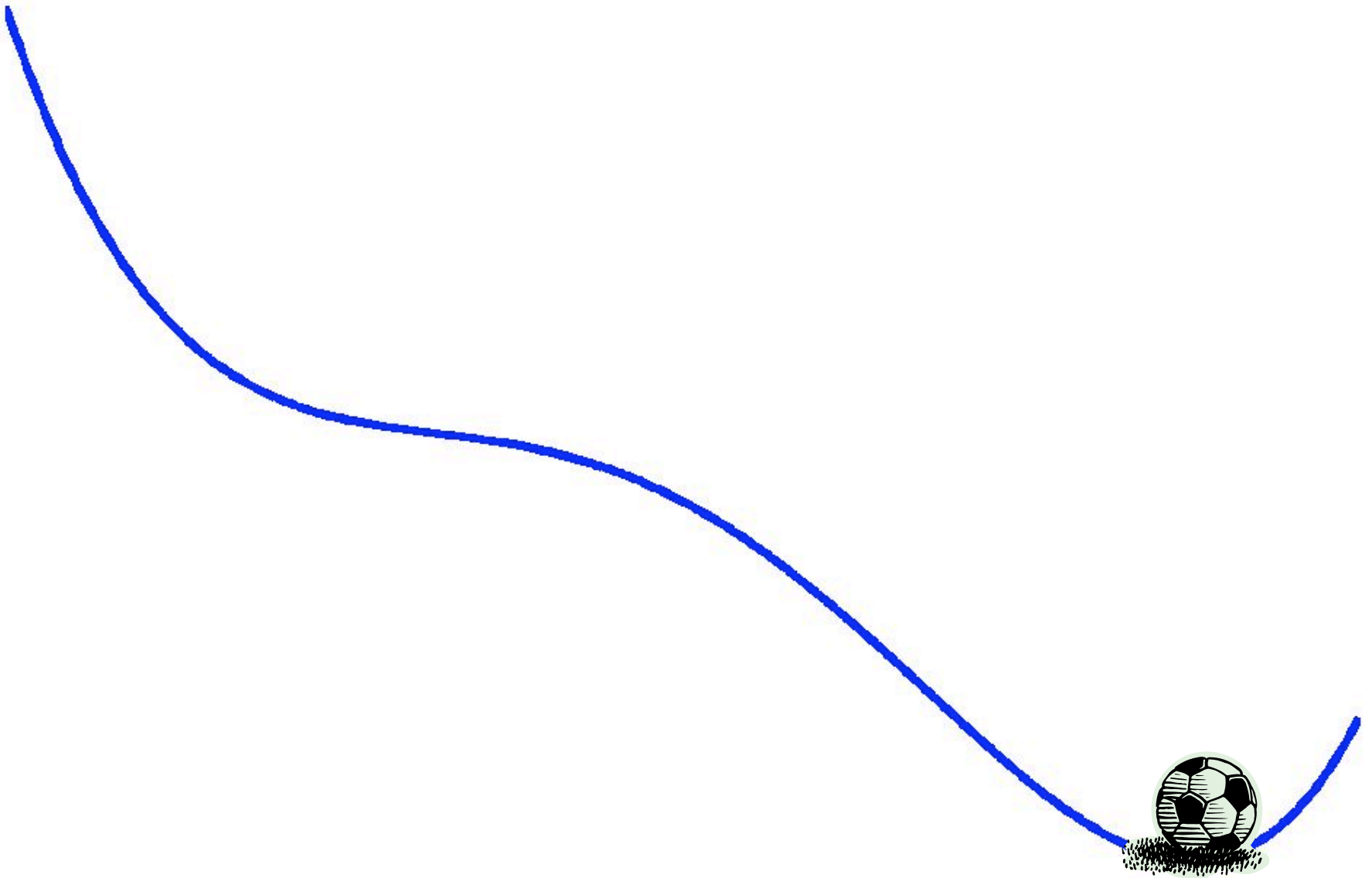
We can change a **stable state** into an **unstable state**



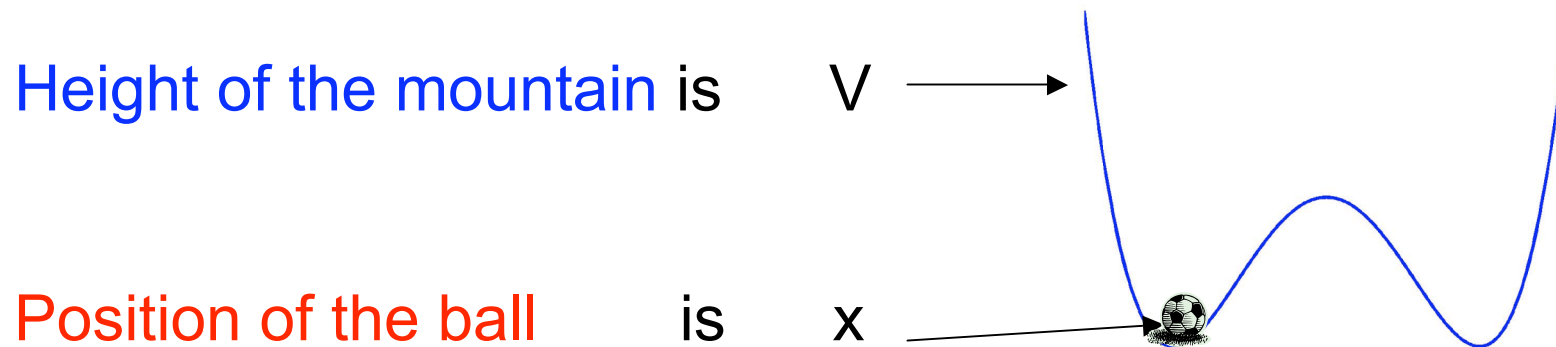








NOW ... do the math!



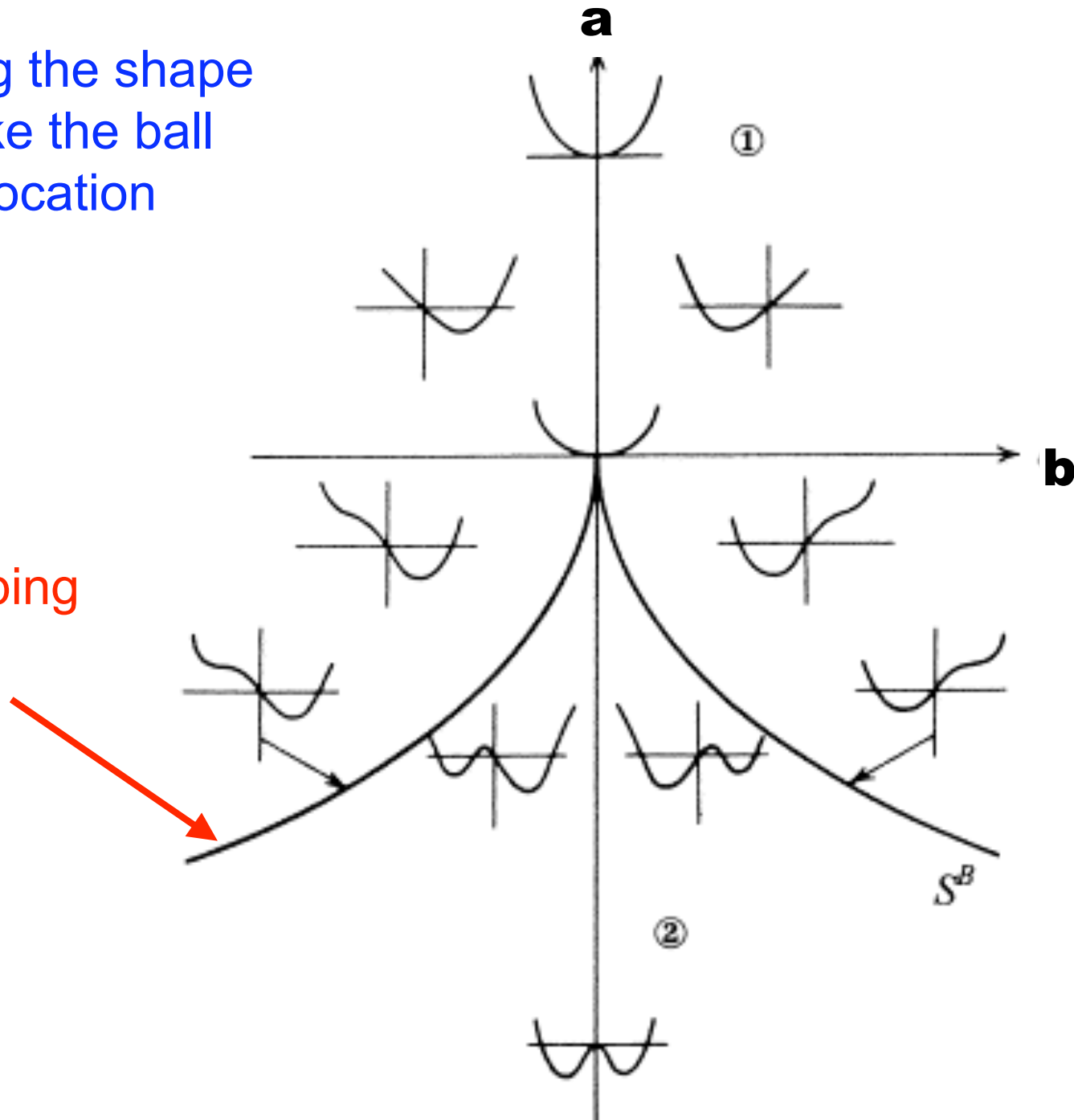
Things that change with time  $a(t)$  and  $b(t)$

$$V = x^4 + a(t)x^2 + b(t)x$$

In the examples  $b(t)$  was changing

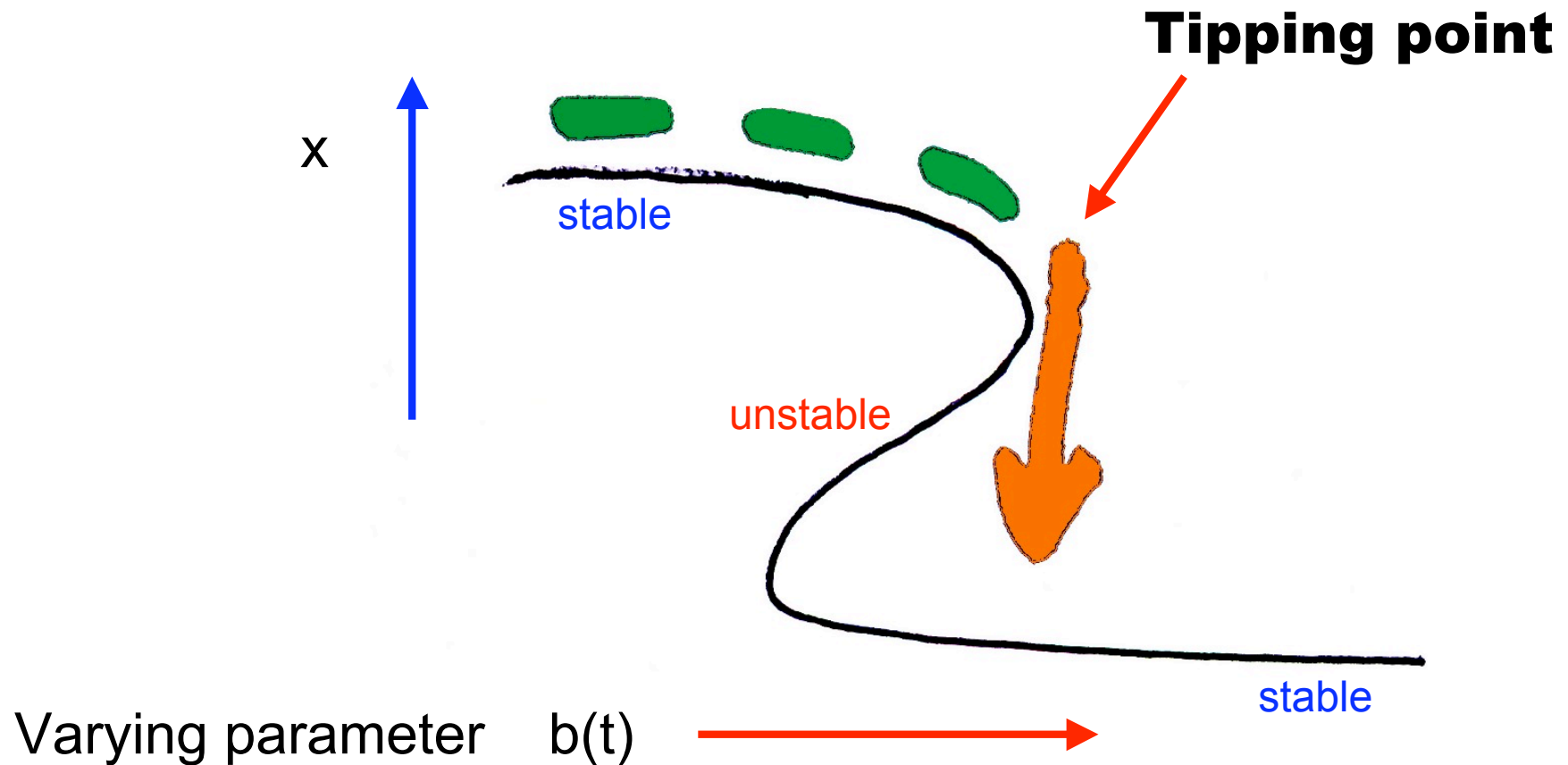
By changing the shape  
we can make the ball  
change its location  
suddenly

Here be tipping  
points



Ball's position  $x$  satisfies the equation

$$3x^3 + 2a(t)x + b(t) = 0$$



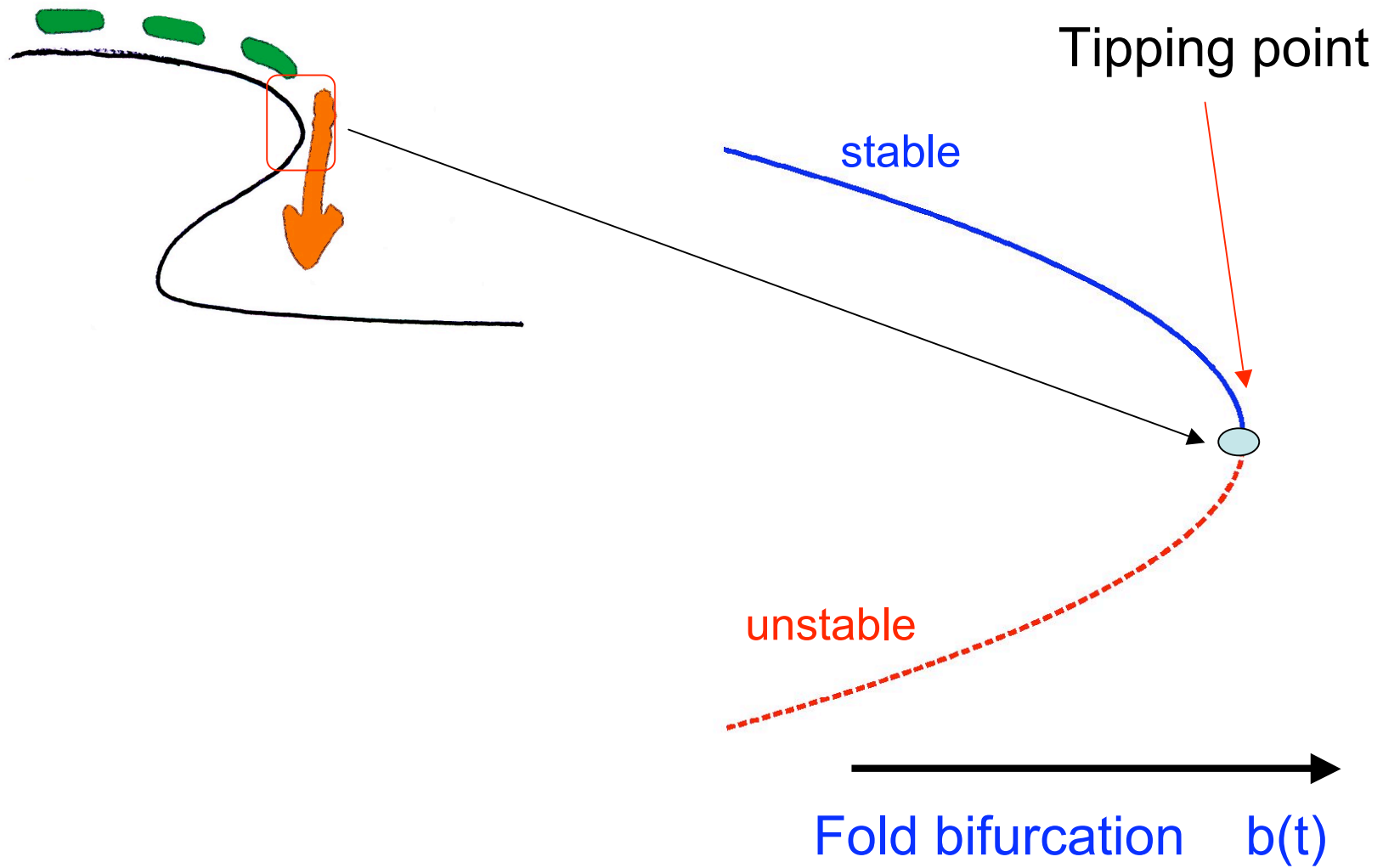


Cam  
OTTAWA CITIZEN  
Caglecartoons.com  
syndicam.com

AS IF HITTING  
AN ICEBERG  
WASN'T  
SURREAL  
ENOUGH...

WORLD ECONOMY

# Close-up to a tipping point

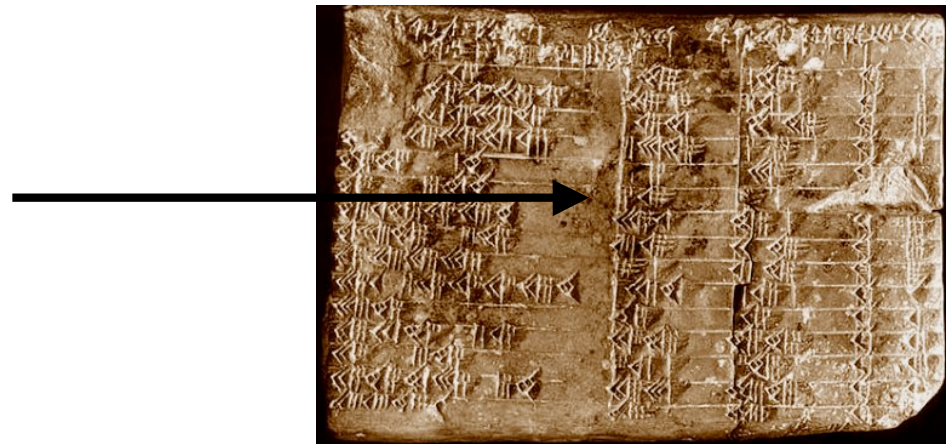


Quadratic equation: archetype of a tipping point

Equilibrium satisfies the **quadratic eqn**

$$b(t) + x^2 = 0$$

Studied by the **Babylonians**





Two roots if  $b$  negative  
or none if  $b$  positive

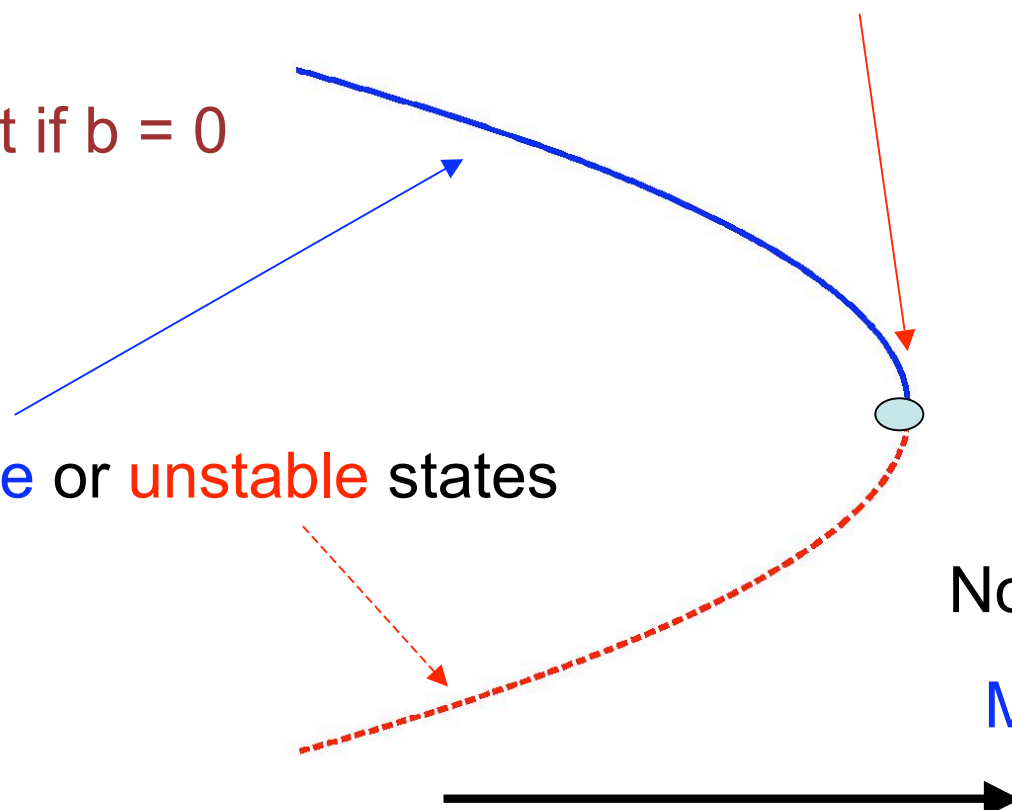
Tipping point if  $b = 0$

Roots: **Stable** or **unstable** states

Tipping point

None:

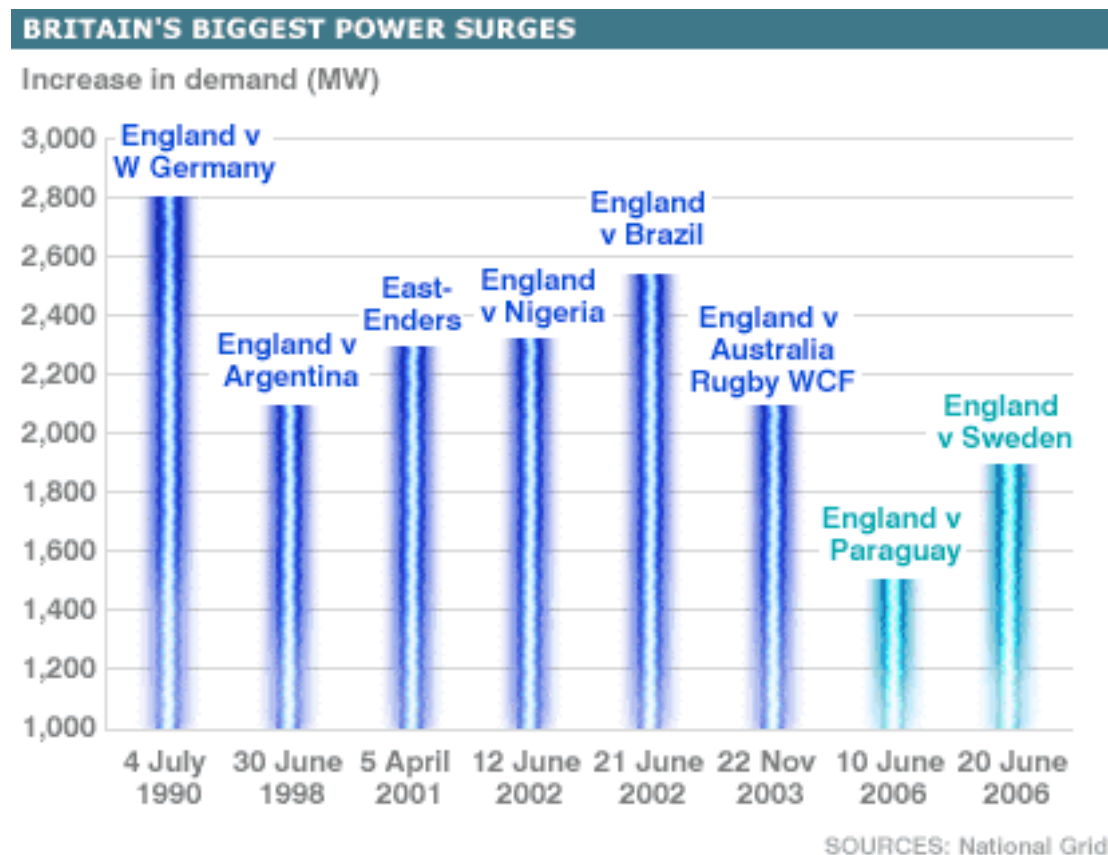
**Major changes**

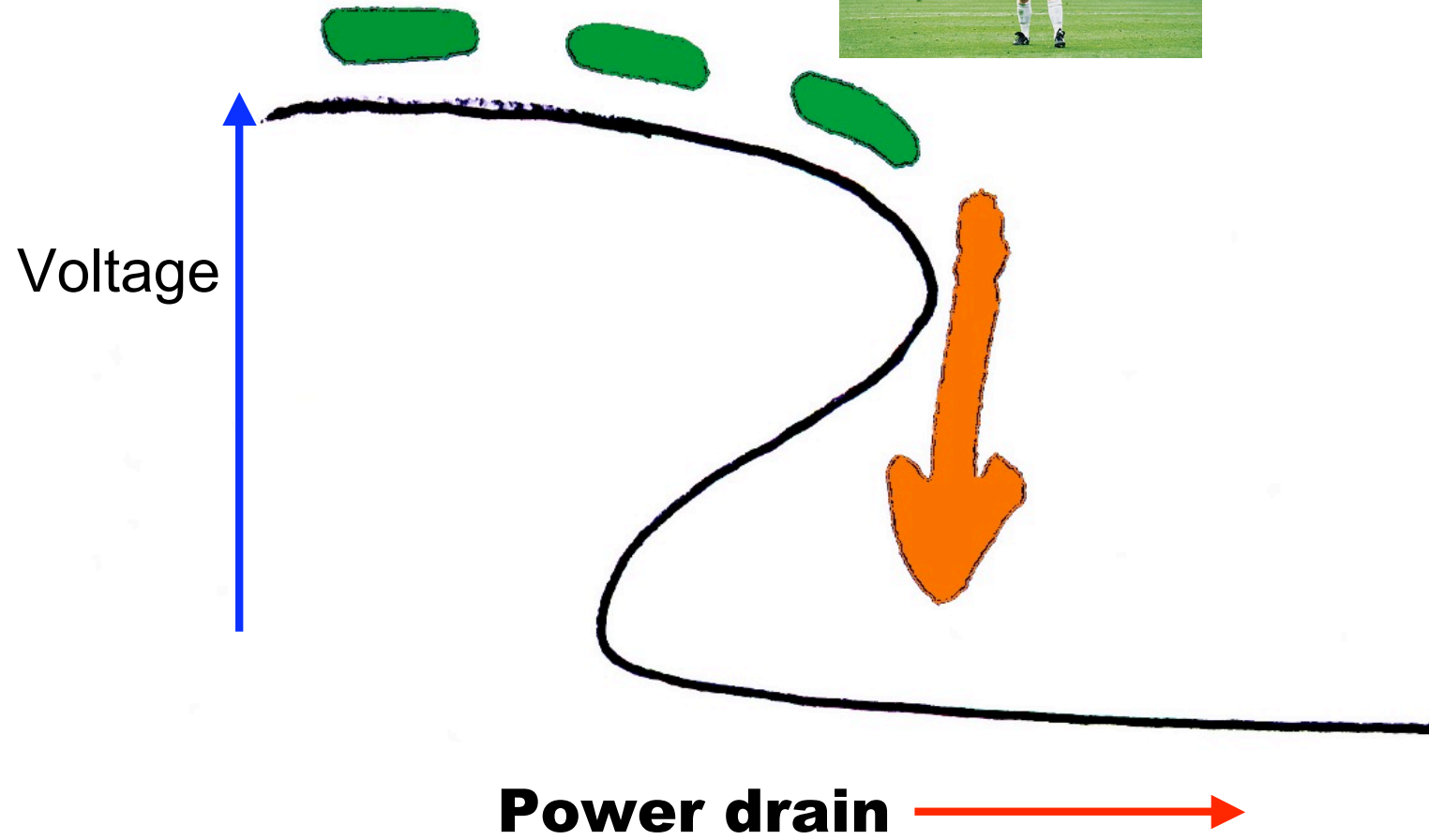


# Example: World Cup 1990: Near Miss!

## England vs. West Germany

At the end of the match, power surged by 2.8 GW  
11% of total capacity or 1 million Kettles





An example from the kitchen: Fluorescent light



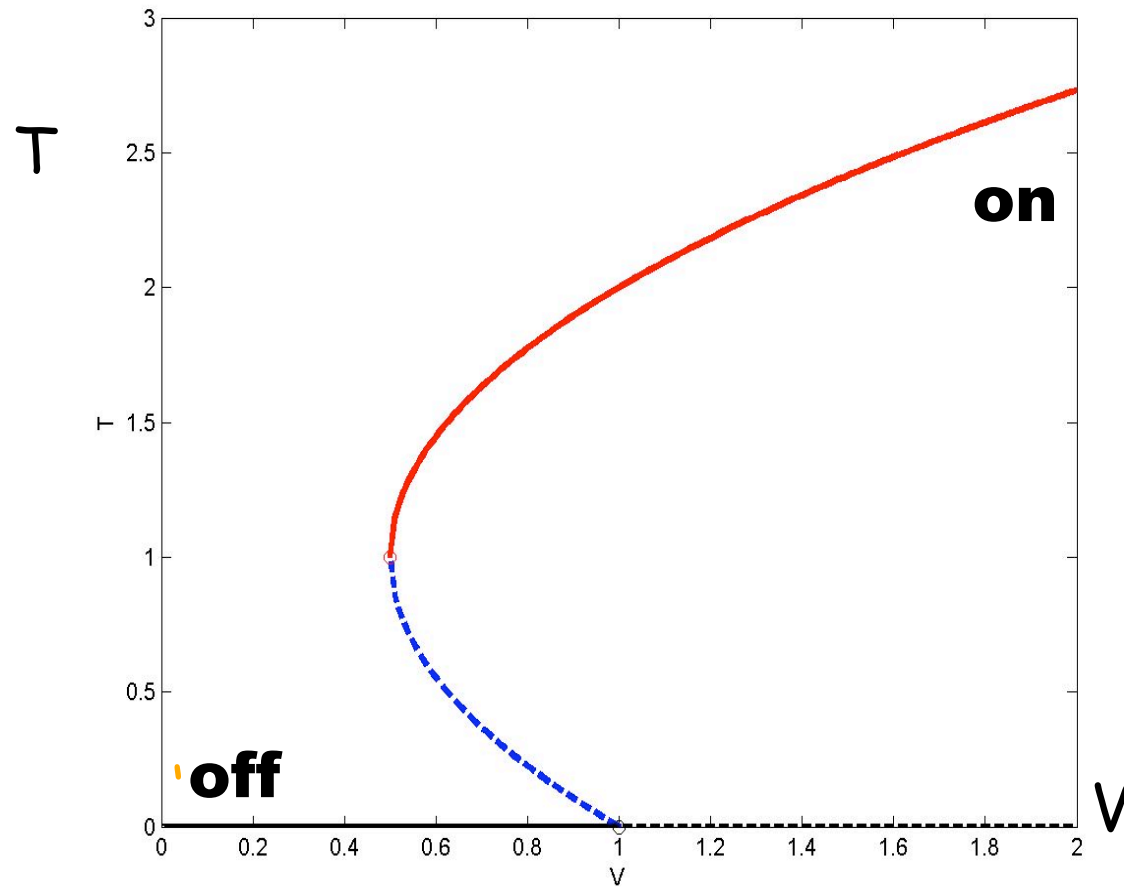
$T_n$  Temperature at each AC cycle

$V$  Applied voltage

$$T_{n+1} = V T_n + T_n^2 - \frac{T_n^3}{2}$$

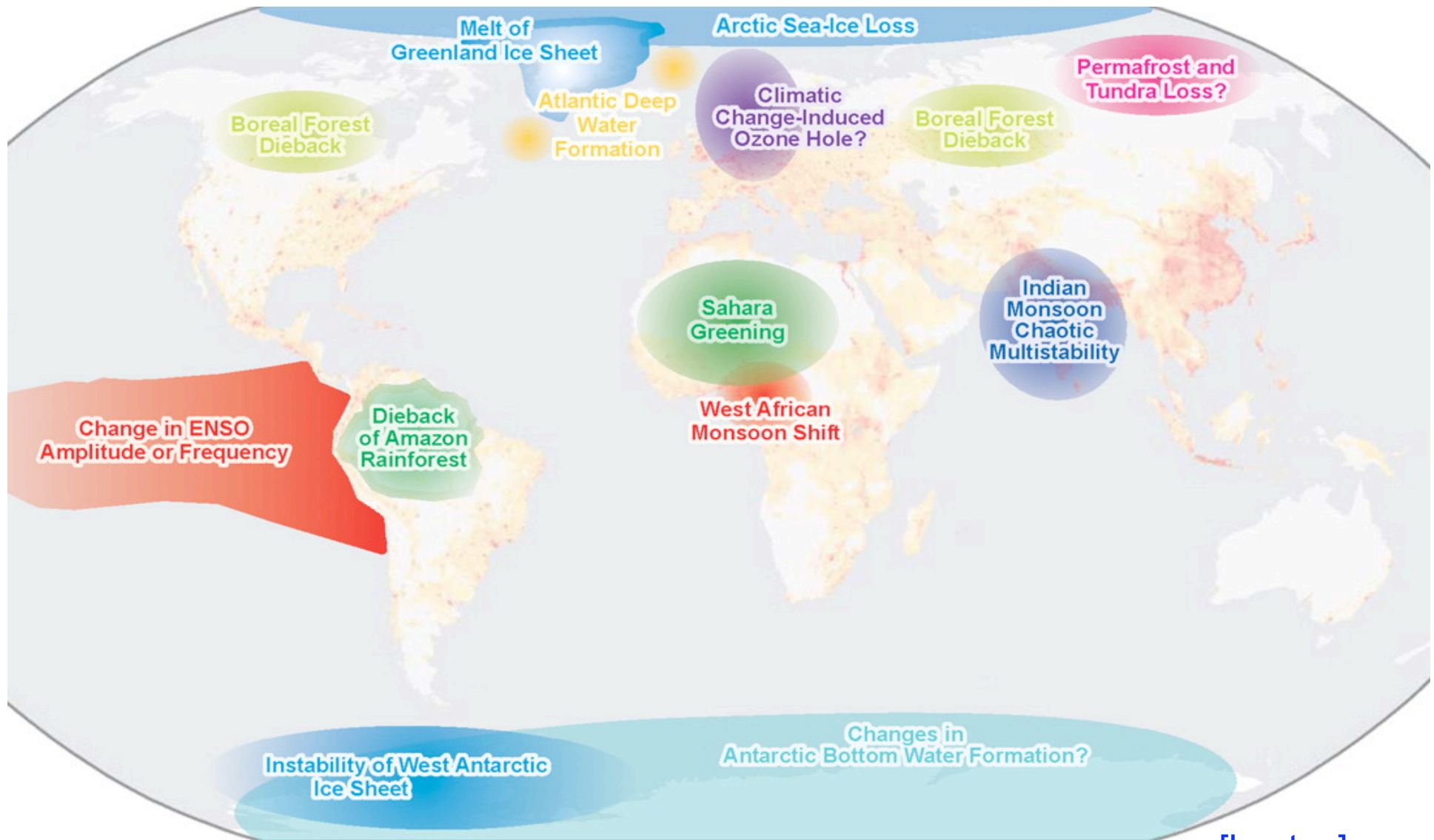
Q. Why do fluorescent tubes need a starter?

$$(V - 1)T + T^2 - \frac{T^3}{2} = 0$$

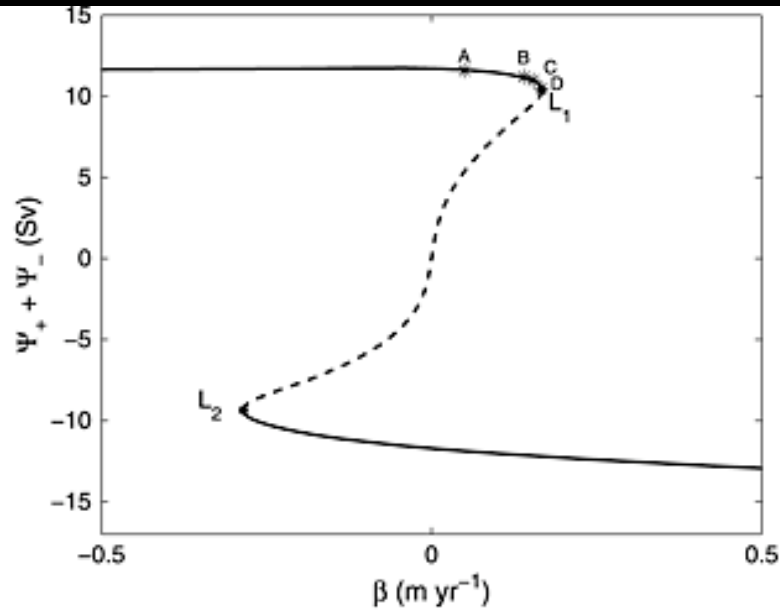
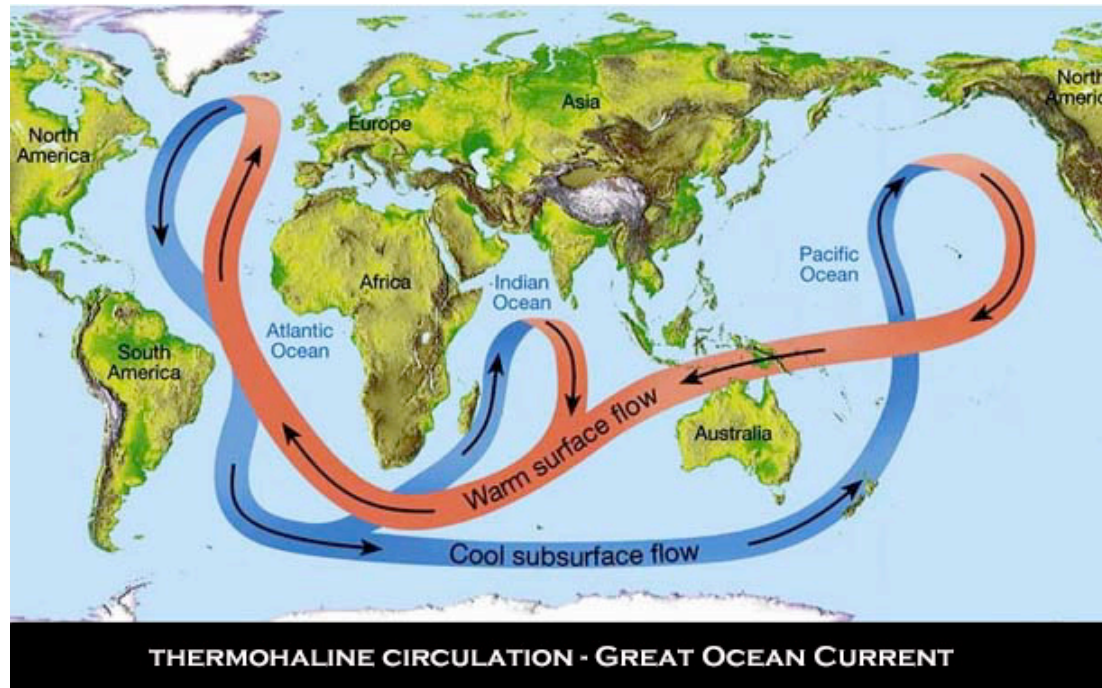


# Potential Tipping Points In The Climate:

What many people are worried about!



# Shut down of the Gulf Stream?



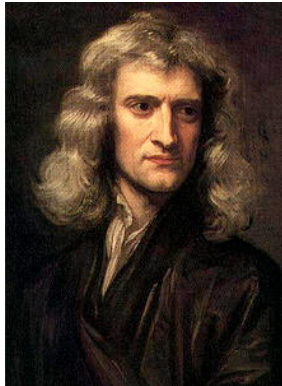
Fresh water flux



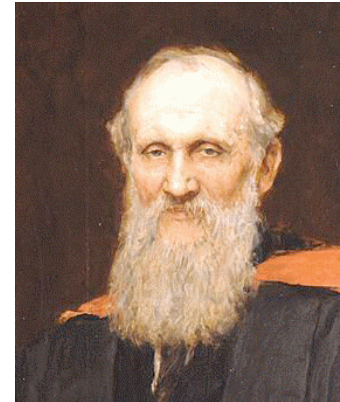
(a) [Dijkstra et. al]

To study these **tipping points** we need **climate models**

Take laws of physics



**Motion**



**Heat**

Turn them into mathematical equations

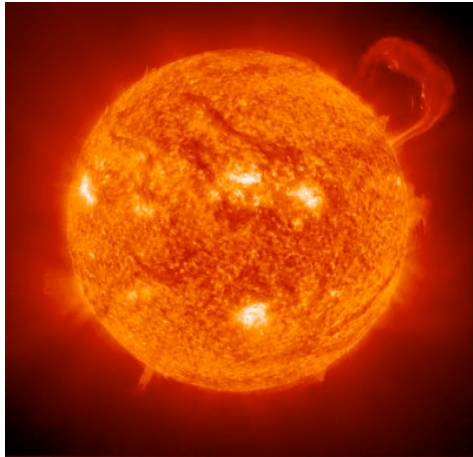
Inform them with data

Solve these on a supercomputer to try to predict the climate





Let's see if we can forecast the climate and ice cover with a simple climate model

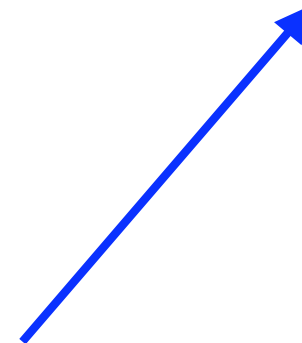


**Heat from Sun:  $S$**

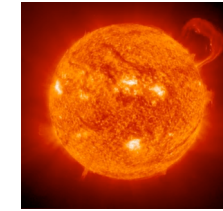


Earth's mean temperature:  $T$

Heat into space



Heat absorbed  $\longrightarrow (1 - a) S$



a Albedo: How well the earth reflects the Sun's rays

Heat radiated away  $\longrightarrow e \sigma T^4$



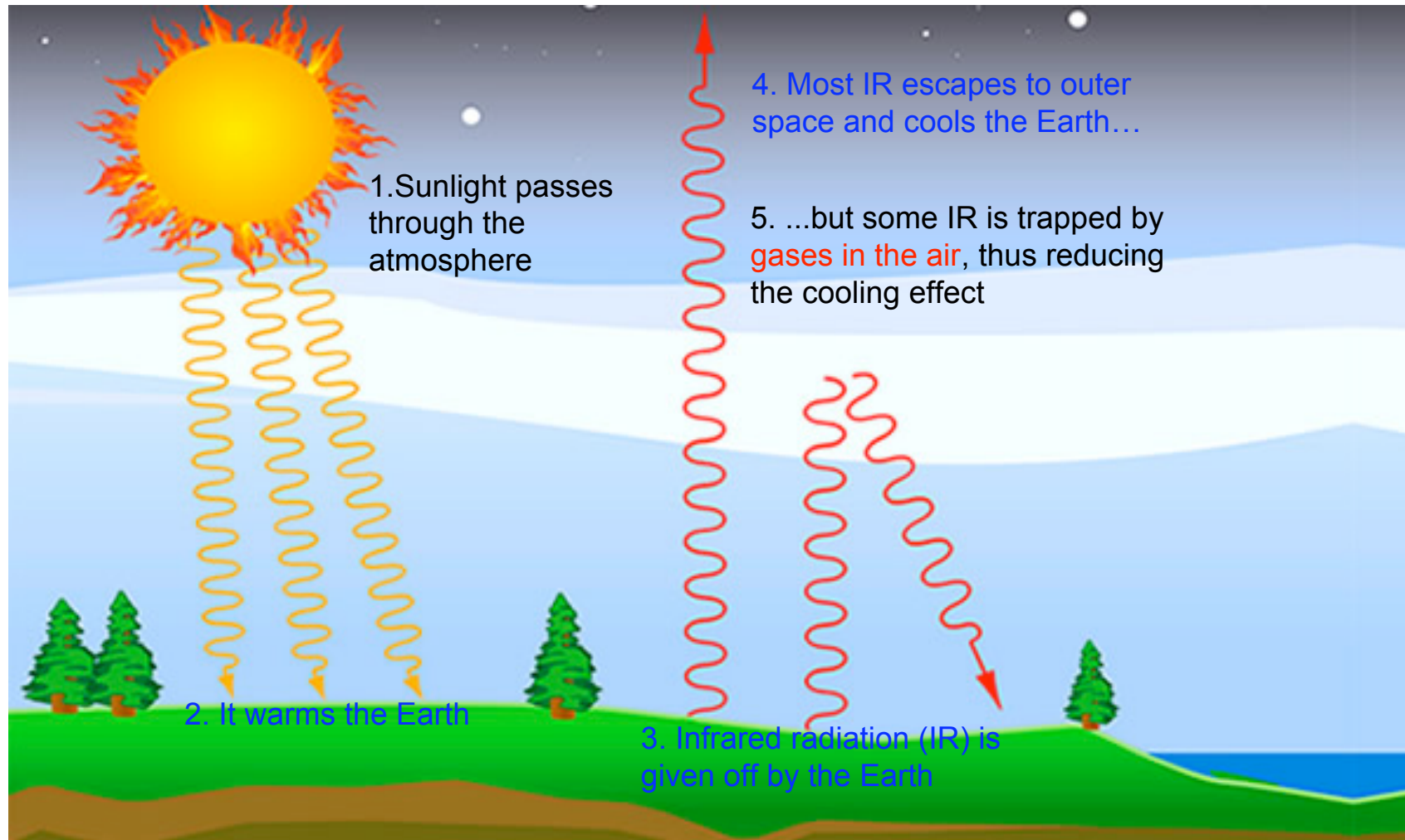
e **emissivity**: How much energy is radiated into space

**Balance these to give a simple climate model**

$$e \sigma T^4 = (1 - a) S$$

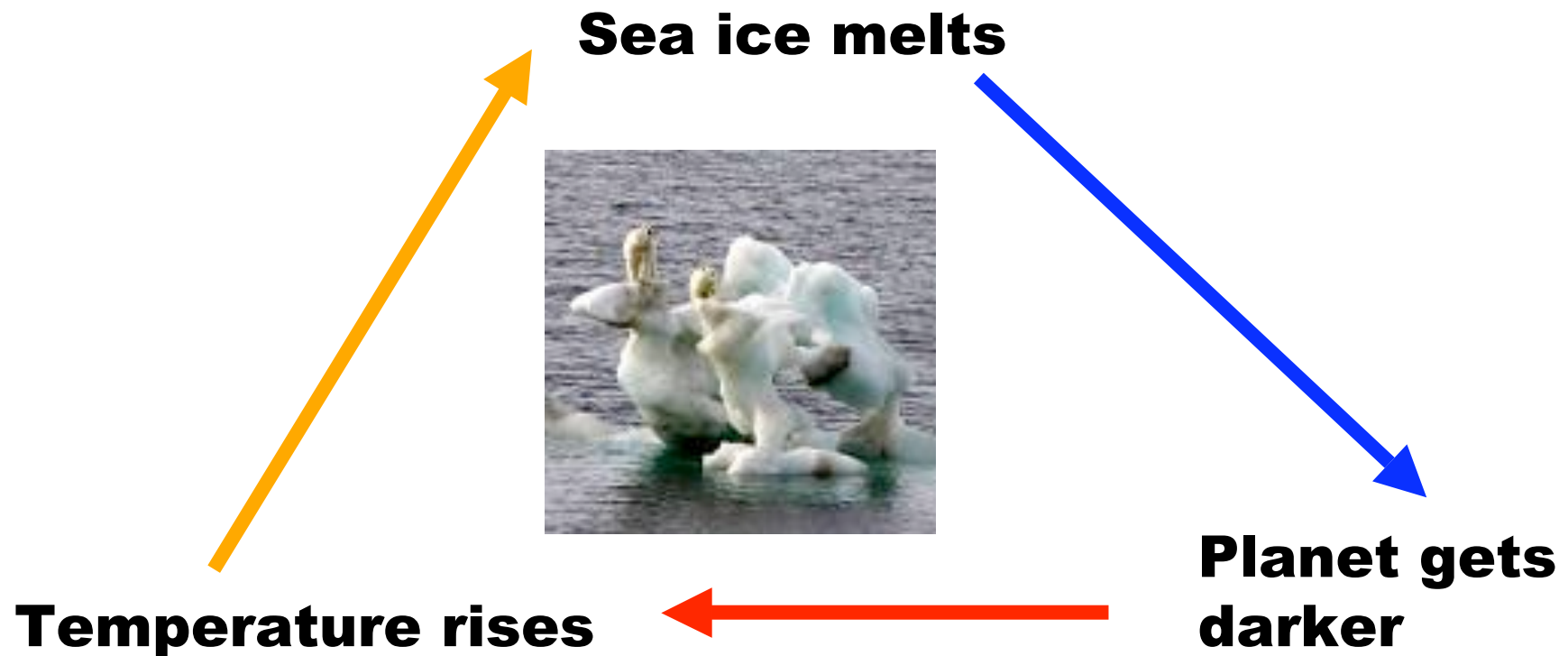
# The greenhouse effect

Emissivity  $e$  decreases as CO<sub>2</sub> increases.  $e$  approx 0.55



Prediction: T increases if a decreases

Leads to the albedo feedback loop

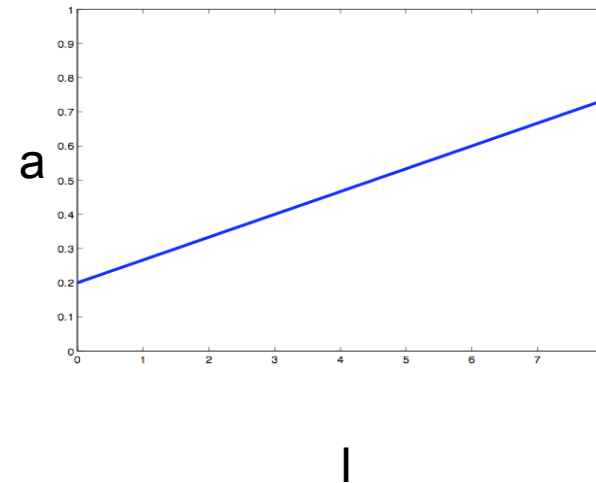


This means that future temperatures may be higher!

**TIPPING POINT??**

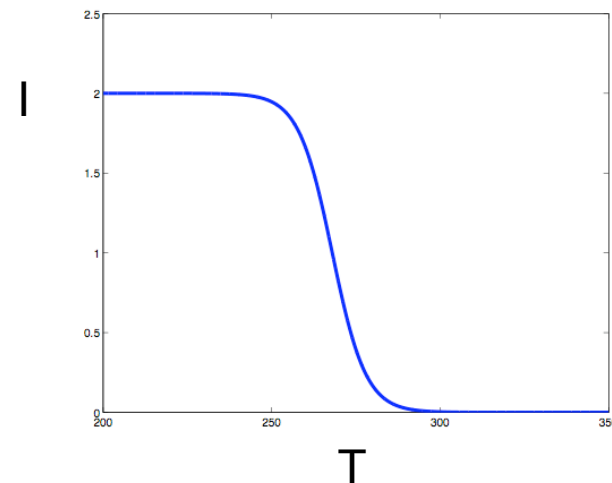
Albedo  $a$  decreases as  
ice content  $I$  decreases

$$a = \alpha + \beta I$$

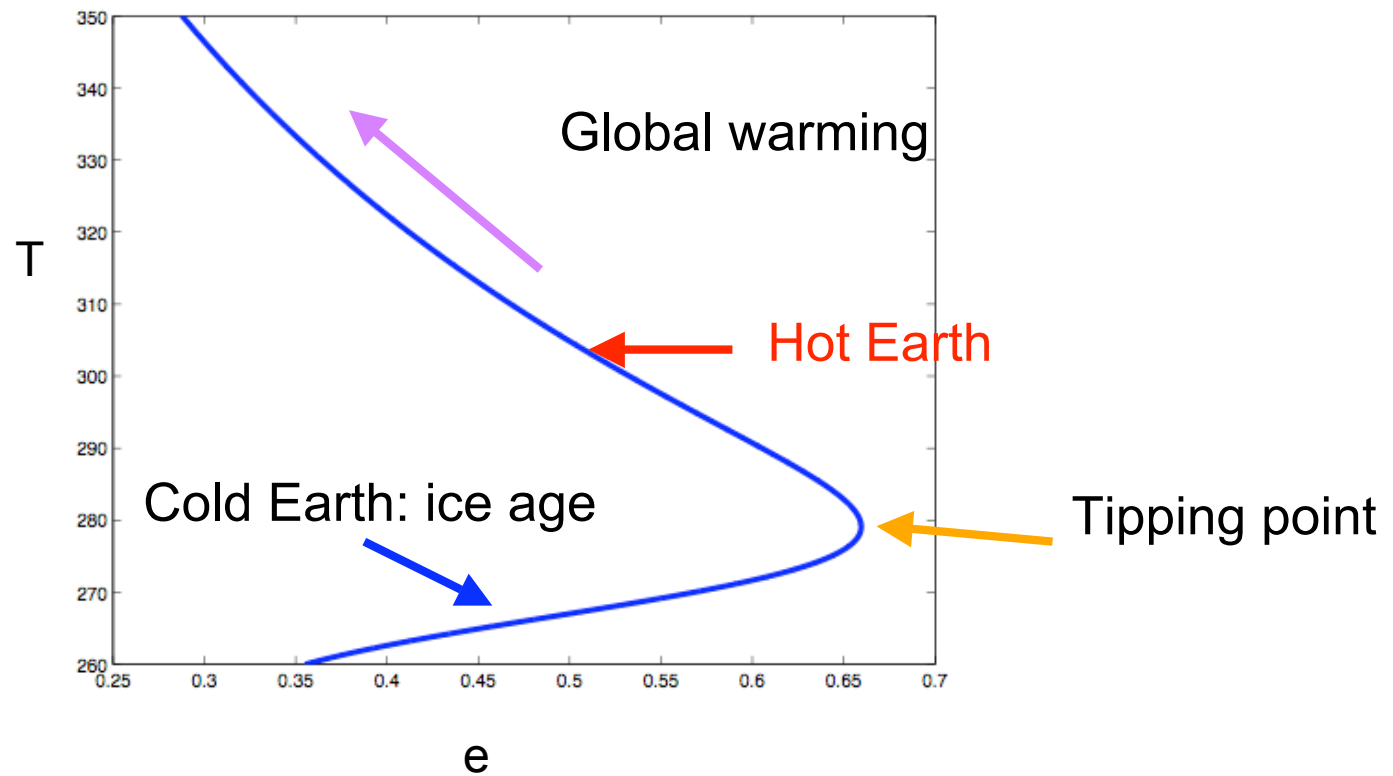


Ice content  $I$  decreases as  
Temp  $T$  increases

$$I = \gamma - \delta \tanh(T - T_0)$$



Putting the three equations together with measured values gives the picture



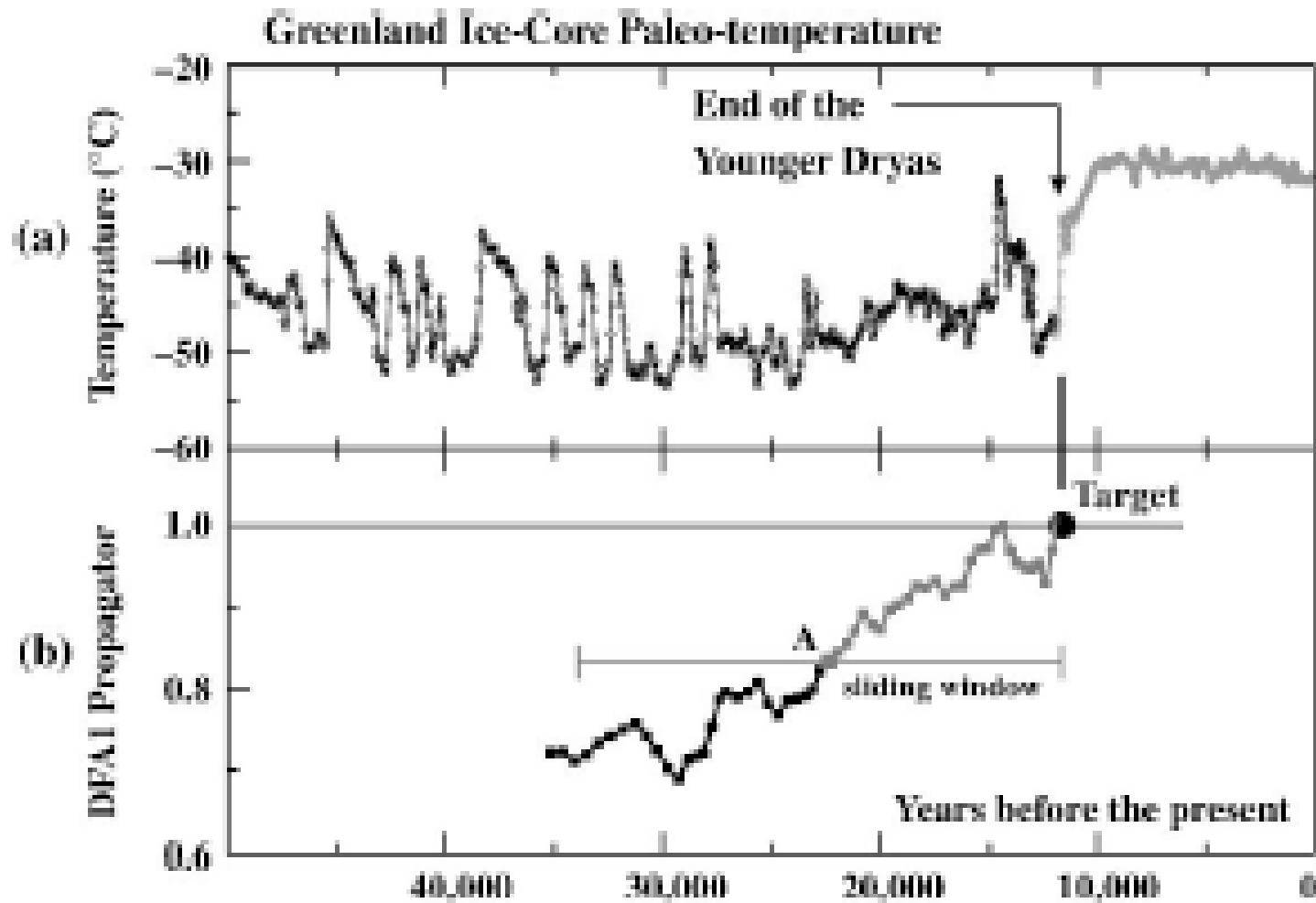
So .. Are we all doomed?



Early warning signs of a tipping point

- **Slower recovery** of the system to small deviations
- Erosion of the **basins of attraction** of the equilibrium states

## Lenton's study of the end of the Younger Dryas Ice Age





# But is every tipping event a fold?

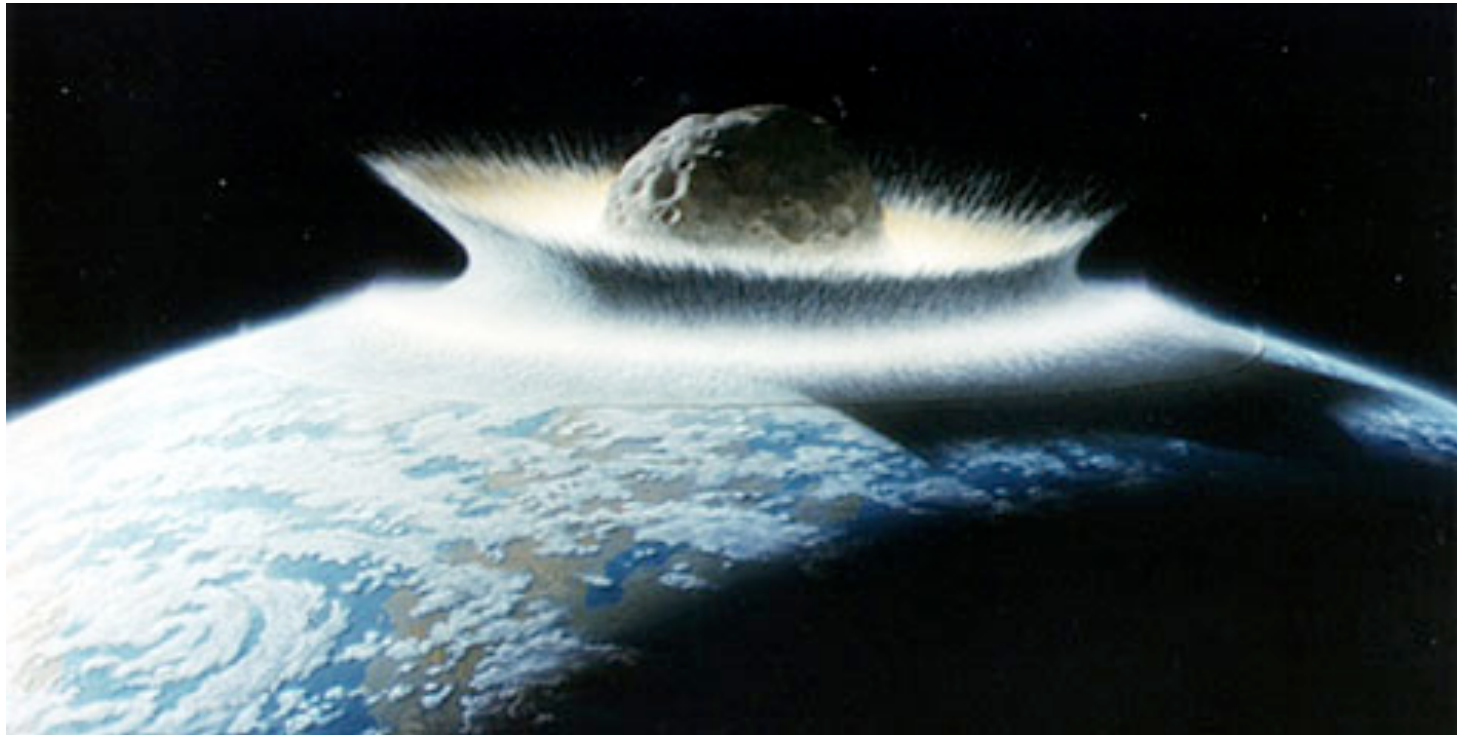
## Earthquakes

Non-smooth bifurcation  
(friction)

MUCH less predictable



Or even .....



## Conclusions

**Do you still want to live in interesting times?**

寧為太平犬，不做亂世人

pinyin: níng wéi tài píng quǎn, bù zuò luàn shì rén

*"It's better to be a dog in a peaceful time than be a man in a chaotic period"*

Maybe you can be an interesting dog!



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