Models of climate: Hot and Snowball Earths

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This work sheet is meant to give you some ideas of the sorts of simple models that are used to give insight into certain aspects of the behaviour of the climate. It will give you some examples of some different climate models and invite you to implement them using **Python**. Whilst the models are very simple, they do give useful insights into the way in which the climate can change as parameters vary, including the effect of tipping points. The models that we are going to be looking at are examples of those used to predict changes in climate over millions of years, and differ from the more precise, but much more complicated models looked at in the other groups, which look at how climate changes in the short term.

Climate models are used to help advise governments. Let's see how well we can do this.

1 *A heat balance model for a steady state climate* The simplest model for climate treats the whole Earth as having a constant and uniform absolute temperature T (measured in degrees Kelvin) which depends upon the incoming solar radiation S (the insolation), the albedo a, and the emmisivity of the atmosphere e. In steady state we have a balance between the incoming radiation $R_i = (1-a)S$ due to the Sun (which is at short wavelengths), and the outgoing radiation $R_o = \sigma e T^4$ from the Earth, which is at long wavelengths. In this balance we have

$$(1-a)S = \sigma e T^4 \tag{1}$$

In the current state of the Earth and its atmosphere we have

$$a = 0.31, \quad e = 0.605, \quad S = 342\ Wm^{-2}, \quad \sigma = 5.67 \times 10^{-8}.$$

(i) Write a Python code to calculate T from equation (1). This code should take as its input a and e and should give T as its output. Check that the answer for T looks sensible.

(ii) If you set $e = 1$ you can calculate the mean temperature of the Moon. Try this and comment on your result

(iii) You can look at the *climate sensitivity* by seeing how small changes in e (due to human activity and the release of Green House gases) affects the temperature T. Make a change of e by 0.1 to see how much T changes.

2. *A time varying heat balance model for climate* The model in Q1 is not very realistic as the Earth's temperature changes. Of course, the temperature of the Earth is not constant, but varies from year to year. A climate model tries to calculate the change in the temperature and to predict the future. To do this we will label the years as $n = 0, 1, 2, 3, \ldots$ and the temperature each year as T_n. Python can do this using a vector T to represent all of these values with component `T[n]` to represent T_n.

In a climate model we try to predict the future temperature T_{n+1} from the present temperature T_n.

We can make the system Q1 more realistic by extending the heat balance model so that the temperature T can evolve according to the formula

$$T_{n+1} - T_n = 0.25\left((1-a)S - \sigma e T_n^4\right), \tag{2}$$

where a, e, σ and S take the values in Q1. Write a Python code to find the values T_n evolving as a function of time in the same manner as you did in in the group exercie. Run this code with a number of different starting values T_0. You should find that the Earth's temperature always returns to the value obtained in Q1. This is reassuring as it implies that the climate is relatively stable.

3 *Hot and cold Earths and the ice-albedo feedback effect* In practice, the albedo of the Earth is not constant, but depends upon the mean temperature T. This is because the albedo depends upon the amount of ice covering the Earth. The more ice there is, the more reflective the Earth is, and thus the higher the albedo. If the Earth is hot then there is less ice and the albedo is low, and if it is cold then there is more ice and the albedo is high. It is estimated that the albedo of a completely ice covered cold Earth (called a snow ball Earth) is about $a = 0.8$, and of a completely ice free hot Earth is about $a = 0.29$. If the albedo is high then there is less energy coming in to warm the Earth, and as a result it is cooler. If the albedo is low, then more energy is available, and it is therefore warmer. This introduces extra sensitivity into the climate so that small changes in e have a larger effect than they did in the model in Q1.

(i) Using these values of a and the values of e, S above, show using your first Python code that the Earth can exist in one of two states, a cold snowball Earth, and a hot Earth. Geological and other evidence exists which implies that the Earth was in such a snow ball state about 600 Million Years ago.

(ii) Use the Python code for Q2 to apply the formula (2) with these new values of a starting from the current temperature of the Earth (which you worked out in Q1). You should find that both the hot and cold states of the Earth are stable. This means that the Earth could exist in either state. Can you explain why using words HINT This is called the ice-albedo feedback effect.

4 *Climate sensitivity and Tipping points* We then must ask the question of how the Earth switched from this snowball state to the current state and whether such switches are likely to occur in the future. In the Budyko-Sellers (1967) model for climate we let the albedo $a(T)$ now be a function of the temperature T. For low values of T the Earth is frozen and the albedo is high, in contrast with high values of T there is no ice and the albedo is low. One example of such a function proposed by Zaliapin and Ghil (2010)

$$a(T) = 0.495 - 0.205\tanh(\kappa(T - T_c)), \tag{3}$$

where $\kappa = 0.133$ and $T_c = 275K$. We will use this expression for the remainder of this worksheet. The function $\tanh(x)$ is given in Python by `np.tanh(x)`.

(i) Using Python, plot $a(T)$ as a function of T for $180K < T < 380K$ (HINT use the `np.linspace` command) . Hence plot the incoming radiation $R_i(T) = (1 - a(T))S$ as a function of T and on the same graph plot the outgoing radiation $R_o(T) = \sigma e T^4$ using the above values of e and σ. Deduce that there are three values of T for which the incoming and outgoing radiation balance. Thus there are three possible climatic states for this value of e corresponding to a cold (snow ball) Earth, a hot Earth (this is our current climate state) and an Earth with an intermediate temperature. We will find that this last state is unstable.

(ii) Take a smaller value of $e = 0.25$ and plot the graphs of R_i and R_o again. Show that in this case there is only one solution of a *hot Earth*. Similarly, show that if $e = 1$ then there can only be a *cold Earth*.

(iii) We can rearrange the energy balance formula to show that at balance we have e given by

$$e = \frac{(1 - a(T))S}{\sigma\, T^4}. \tag{4}$$

Taking $220K < T < 350K$, use Python to calculate e from (4) and then plot T as a function of e using `plt.plot(e,T)`. You should get an 'S-shaped' curve with two values of e at which there is a *tipping point* close to which we see a sudden change from a cold to a hot earth or vice-versa associated with a small change in the value of e. Comment on this from the view point of climate change. Is this a reasonable mechanism for escaping from a snowball Earth?

(iv) [Advanced for the ambitious: This needs you to solve a nonlinear equation] Taking $e = 0.605$ as before, estimate the corresponding value of T for a hot Earth (the current state of the climate). See how sensitive this estimate is to small changes in e. This estimate should be more sensitive than the one in Q1 due the amplifying effects of the ice-albedo effect. It is calculations such as these which the IPCC use to determine current climate sensitivity to small changes in the level of Greenhouse gases.

5 Modify your Python code so that the temperature changes according to the equation

$$T_{n+1} - T_n = 0.25\left((1 - a(T_n))S - \sigma e T_n^4\right). \tag{5}$$

For each of the cases $e = 0.605, e = 0.25, e = 1$, experiment with a range of different initial values of T to find which initial condition evolves to a hot or a cold Earth.

6 Challenge *Passing through a tipping point* The value of e varies with time. In the past this could have been due to volcanic activity or the release of green house gases from the oceans. Now it is due to the effects of burning fossil fuels and/or the release of Methane as perma-frost melts. Extend your Python code so that it changes e slowly, and T_n changes with it according to the formula

$$T_{n+1} - T_n = 0.25\left((1 - a(T_n))S - \sigma e_n T_n^4\right), \quad e_n = e_0 + \lambda n \tag{6}$$

For example, take $e_0 = 0.5, \lambda = 0.1$ or $e_0 = 0.5, \lambda = -0.1$. Now find the evolution of T taking $T_0 = 273$. Plot T as a function of n and also T as a function of e. Comment on your results.

7. This exercise has introduced you to a simple energy balance model with feedback. Its simplicity means that we can run it to simulate the climate over long periods, not just the relatively short time-scales used in the highly complex models used by the IPCC. Do you think that any of these models we have considered are indeed reasonable for simulating the long term evolution of climate. List their advantages and disadvantages. How could the models be improved? How can they be tested? These are the sort of issues that theoretical modellers (such as ourselves) have to consider in our research.

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