RI Masterclass Exercise Sheet

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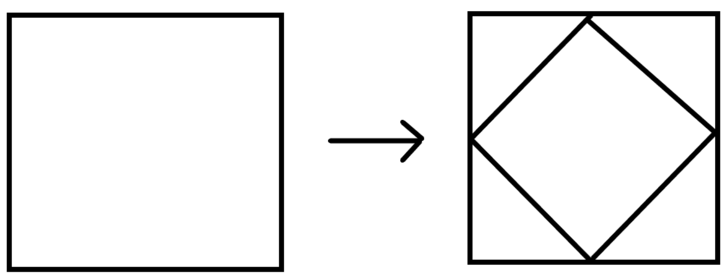
Exercise 1

Start by drawing a square. Then (as in recursion), apply the following operation three times to the figure you obtained in the previous step:

1. For each square, locate the midpoint of each site.
2. Connect the midpoints as follows:

1.png


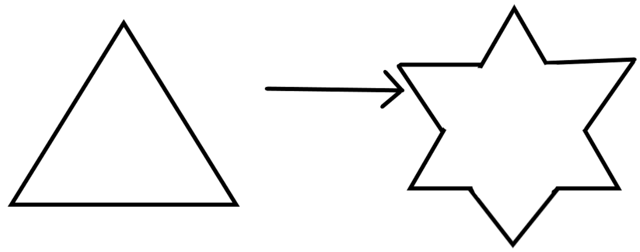
How many triangles are there in the final figure?

Exercise 2

Let's produce a second figure using recursion. This time we start with an equilateral triangle. For each side we perform the following three steps:

1. Divide the site into three equal parts
2. Construct an equilateral triangle which has the second part as one side
3. Remove the second part from the original side

The following image sketches the original equilateral triangle (left) and the figure we obtain after applying the operation once (right):



Draw a sketch of the figure we get when we apply this operation again to the figure we obtained in the previous step. How many edges does the figure have after we applied this operation three times (with the right plot being the result when applying the operation once)?

Exercise 3

We introduced the Fibonacci numbers as the numbers we obtain when starting with $a_0 = 0$ and $a_1 = 1$ and then defining $a_2 = a_0 + a_1$, $a_3 = a_1 + a_2$ and so on. We also saw that we can create a Fibonacci spiral (that looks a bit like the shell of a snail).

Suppose we started with $a_0 = 2$ and $a_1 = 2$ instead of $a_0 = 0$ and $a_1 = 1$, but the operation stays the same.

a) What are the next five values a_2, a_3, a_4, a_5 and a_6 in the sequence?

b) Would we still be able to create a spiral from the sequence of values we get with these new starting values?

Exercise 4

We saw how the dragon curve can be constructed by unfolding a strip of paper. What would the curve look like if you keep all the 'crests' at 90°, but straighten all the 'crests'?

To do this exercise, you first need to reconstruct the dragon curve sequence - you can do it either by recursion, or by plaing with your own paper strip.

Exercise 5

In this exercise, we will generate a close cousin of the dragon curve. However, this time we will skip playing with paper and go straight to maths.

We start by writing the first sequence

$$L \;\; R \;\; R \;\; L.$$

Then, on both ends, as well as between every two letters, we add the sequence '$LRRL$'. It looks like this:

$$LRRL\ L\ LRRL\ R\ LRRL\ R\ LRRL\ L\ LRRL.$$

By interpreting L as a left turn by 90° and R as a right turn by 90°, draw the curve. What does it resemble?

If there is time, you can generate a longer string and draw a longer curve by inserting '$LRRL'$ between any two letters again. How long would it be?