DE LA RECHERCHE À L'INDUSTRIE

Particle transport in stochastic media: an overview and some open questions

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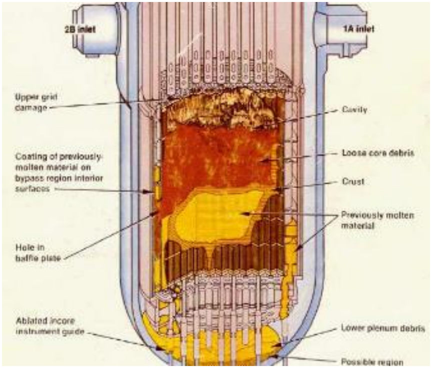
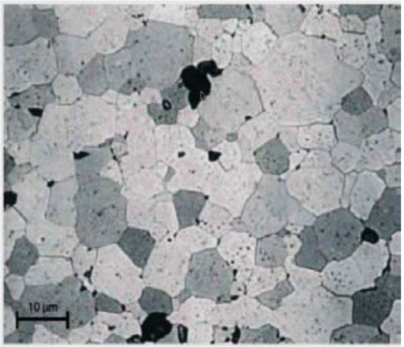
Fuel degradation: melting and solidification



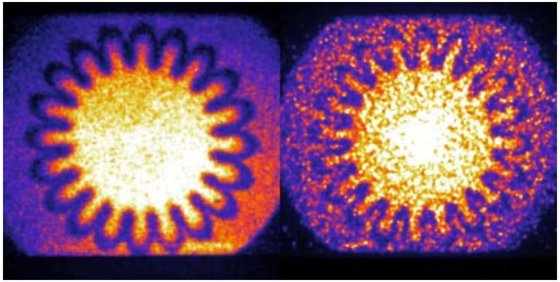
Three Mile Island accident

Dispersion of grains in fuel pellets



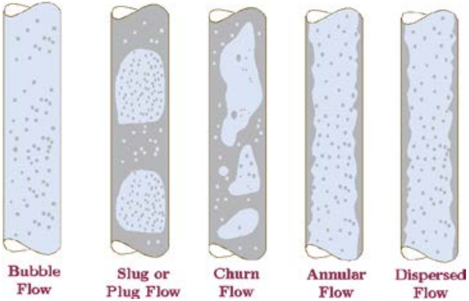
Micrography of a MOX pellet

Turbulent layers in inertial confinement fusion



Rayleigh-Taylor instabilities

Steam-water mixtures



Adiabatic flow through a pipe

- **Assumption:** ∃ a collection of states (physical realizations) $X = \{q\}$

  with a probability density function $\mathcal{P}(q)$

  $q(\mathbf{r})$: mapping between material properties and the spatial point $\mathbf{r}$

  $\Sigma_t^{(q)}(\mathbf{r})$, $\Sigma_s^{(q)}(\mathbf{\Omega}' \to \mathbf{\Omega}, \mathbf{r})$ and $Q^{(q)}(\mathbf{r}, \mathbf{\Omega})$: q-dependent material properties and source

- **Goal:** compute the **ensemble-averaged angular flux**

$$\langle \varphi(\mathbf{r}, \mathbf{\Omega}) \rangle = \int \mathcal{P}(q) \varphi^{(q)}(\mathbf{r}, \mathbf{\Omega}) dq$$

❖ Pomraning (1991)

Where $\varphi^{(q)}$ solves the **linear Boltzmann equation** *for a given realization q*:

$$\mathbf{\Omega} \cdot \nabla \varphi^{(q)}(\mathbf{r}, \mathbf{\Omega}) + \Sigma_t^{(q)}(\mathbf{r}) \varphi^{(q)}(\mathbf{r}, \mathbf{\Omega}) = \int \Sigma_s^{(q)}(\mathbf{\Omega}' \to \mathbf{\Omega}, \mathbf{r}) \varphi^{(q)}(\mathbf{r}, \mathbf{\Omega}') d\mathbf{\Omega}' + Q^{(q)}(\mathbf{r}, \mathbf{\Omega})$$

QUENCHED VS. ANNEALED DISORDER

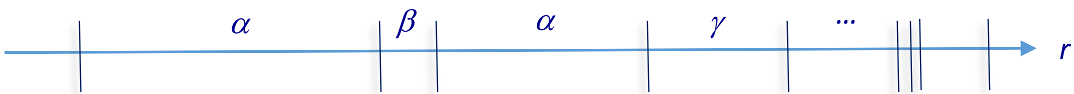
- **Quenched disorder** approach
  - Generate an *ensemble* of N realizations from $P(q)$
  - Solve the transport equation $B^{(q)}\varphi^{(q)} = Q^{(q)}$ for each realization q
  - Deduce the ensemble-averaged flux $\langle\varphi(\boldsymbol{r},\boldsymbol{\Omega})\rangle \simeq \frac{1}{N}\sum_q \varphi^{(q)}(\boldsymbol{r},\boldsymbol{\Omega})$
  - **Advantage:** provides **reference** solutions for particle transport
  - **Drawback: computational burden**

- **Annealed disorder** approach
  - Write a transport equation for $\psi = \langle\varphi(\boldsymbol{r},\boldsymbol{\Omega})\rangle$, which in general is not closed
  - Apply a *closure formula* and obtain an "effective" transport equation $B^*\psi = Q^*$
  - **Advantage: reduced computer time**
  - **Drawback: approximate** method

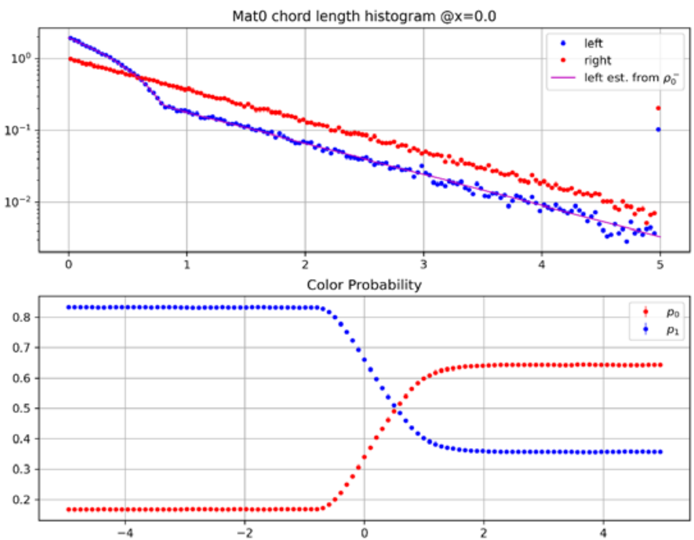
MARKOV MEDIA: ONE-DIMENSIONAL CASE

- One-dimensional *n-nary* **Markov media** generated by transition rates $\rho_{\alpha,\beta}(r,\pm)$

$$\{X(r)\} = \alpha \text{ at position } r \text{ describes a Markov chain among discrete states}$$



Markov jump process



- "Material length" distribution: non-homogeneous exponential with parameter

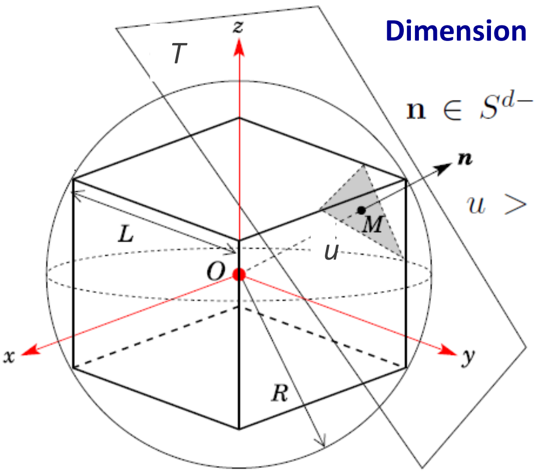
$$\rho_\alpha(r,\pm) = \sum_{\beta \neq \alpha} \rho_{\alpha,\beta}(r,\pm)$$

- Chapman-Kolmogorov equation:

$$\frac{\partial}{\partial r} p_\alpha(r) = \sum_{\beta \neq \alpha} p_\beta(r)\rho_{\beta,\alpha}(r,\pm) - p_\alpha(r)\rho_\alpha(r,\pm)$$

Poisson hyperplane process

$$T(\mathbf{n}, u) = \{\mathbf{x} \in \mathbb{R}^d : \langle \mathbf{n}, \mathbf{x} \rangle = u\}$$



Dimension d

$$\mathbf{n} \in S^{d-1}$$

$$u > 0$$

- ❑ Poisson distribution for the parameters *u* and ***n***, with density

$$f(\mathbf{n}, u) : S^{d-1} \times (0, +\infty) \to [0, \infty)$$

- ❖ Schneider & Weil (2008)

- ❖ Chord distribution at **r,Ω** (Markov):

$$\mathcal{P}(\ell|\mathbf{r}, \mathbf{\Omega}) = \rho(\mathbf{r} + \ell\mathbf{\Omega}, \mathbf{\Omega}) e^{-\int_0^\ell ds \rho(\mathbf{r} + s\mathbf{\Omega}, \mathbf{\Omega})}$$

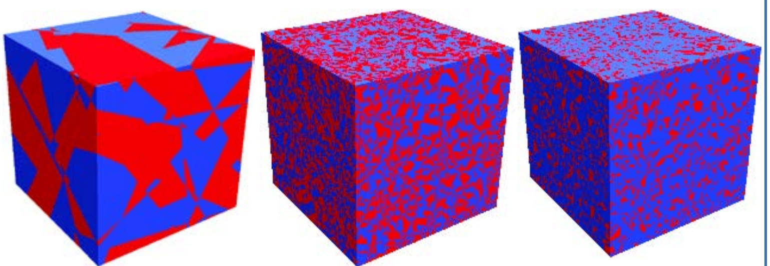
with:

$$\rho(\mathbf{r}, \mathbf{\Omega}) = \int_{S^{d-1}} |\langle \mathbf{\Omega}, \mathbf{n} \rangle| f(\mathbf{n}, \langle \mathbf{n}, \mathbf{r} \rangle) d\mathbf{n}$$

- ❖ Switzer's **coloring** procedure: color each cell of the Poisson tessellation with probability p_α

$$\rho_\alpha(\mathbf{r}, \mathbf{\Omega}) = (1 - p_\alpha)\rho(\mathbf{r}, \mathbf{\Omega})$$

  - Larmier et al. (2016)

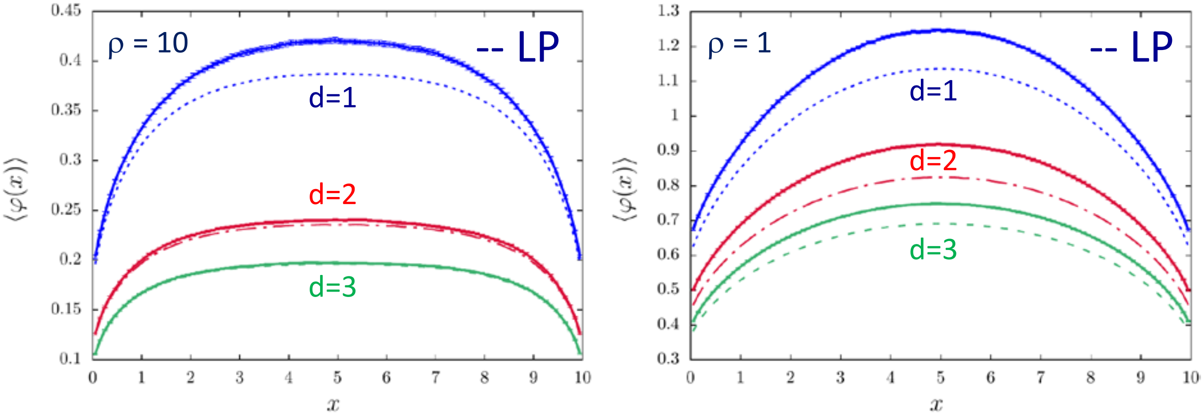


- ➢ Any **other ways** of « building » Markov media?

- By introducing the **Levermore-Pomraning (LP) closure**, it is possible to derive an *effective transport equation* for the material-averaged flux $\psi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega}) = p_{\alpha}\,\langle\varphi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega})\rangle$

$$\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\,\psi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega}) + \Sigma_t^{\alpha}\,\psi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega}) = \underbrace{\int \Sigma_s^{\alpha}(\boldsymbol{\Omega}'\to\boldsymbol{\Omega})\,\psi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega}')d\boldsymbol{\Omega}' + Q^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega})}_{\text{Regular Boltzmann equation for « species » }\alpha} \underbrace{+ \rho_{\beta}\psi^{\beta}(\boldsymbol{r},\boldsymbol{\Omega}) - \rho_{\alpha}\psi^{\alpha}(\boldsymbol{r},\boldsymbol{\Omega})}_{\textbf{Coupling}\text{ term}}$$

- **Comparison** with respect to reference solutions:



➢ Can we build **robust** annealed-disorder transport equations?

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Thanks for your attention

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