Neutron Transport Days 2021

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Branching Structures Meeting
Bath, September 17, 2021

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Background:

Large deviations

Monte Carlo methods

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Stochastic control ↔ PDEs and variational problems

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Stochastic control ⟷ PDEs and variational problems

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Importance sampling

MCMC

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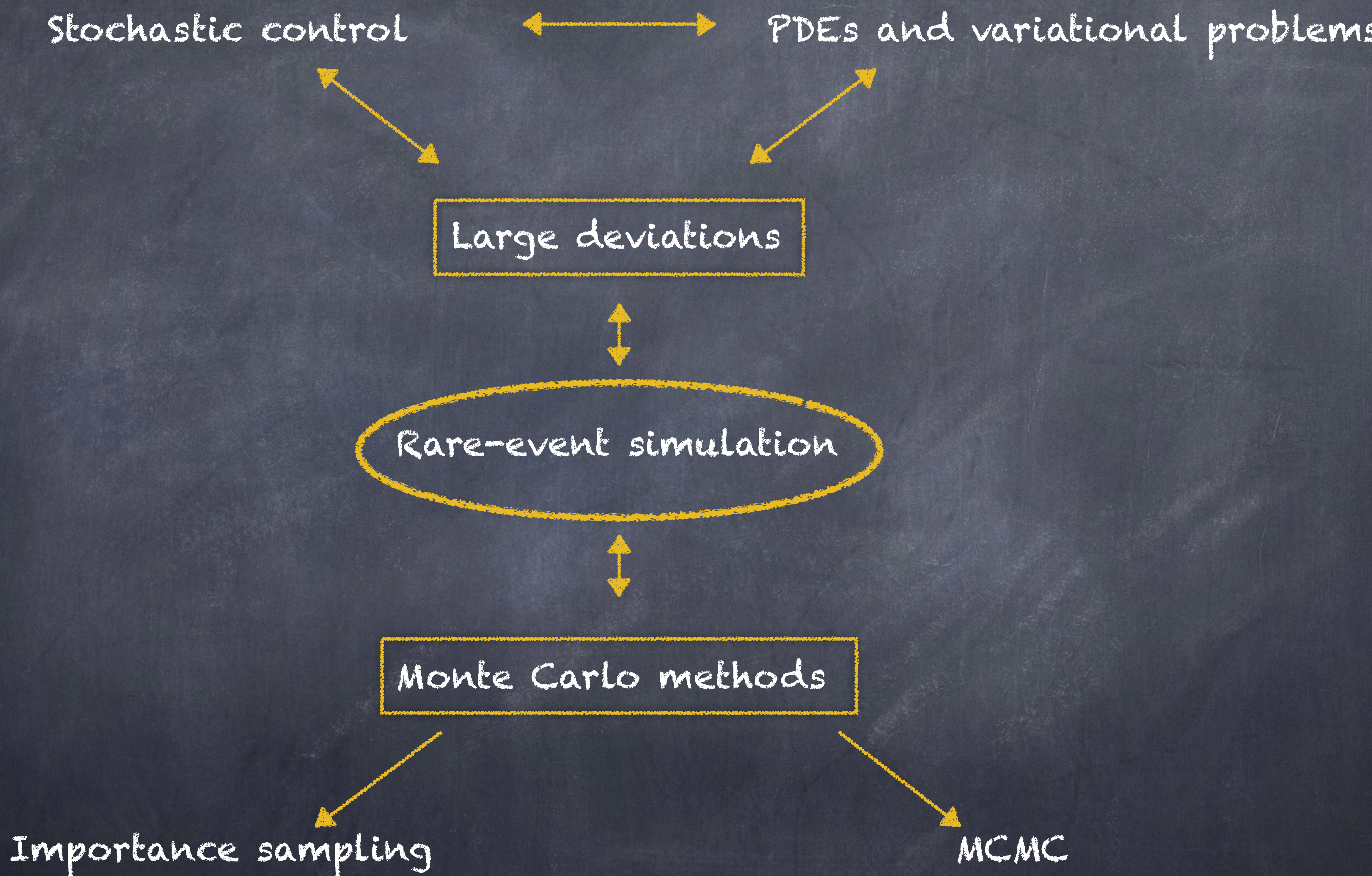
Stochastic control
PDEs and variational problems
Large deviations
Rare-event simulation
Monte Carlo methods
Importance sampling
MCMC

[From "Monte Carlo methods for the NTE" (Cox, Harris, Kyprianou, Wang)]

9. Discussion and numerical comparison of methods.

9.1. *Towards Improved Monte-Carlo Methods for the NTE.* The results of the previous section (in particular Remark 8.1) suggests that extremely efficient methods for estimating λ_* can be developed if the eigenfunction φ is known. Unfortunately, finding the optimal φ is generally at least as hard as finding the optimal λ_*, so this is not immediately helpful. However, importance sampling can still be highly effective in reducing the variance of the numerical scheme even when the function φ is not known. The key question is how to construct informative functions h which might approximate φ well.

The benefits of combining SMC methods with importance sampling has been investigated by (e.g. [29]). It is in this combination that we expect the results in Sections 7 and 8 to be of most benefit. Specifically, we would anticipate using SMC methods to sample/resample from a population of particles, which themselves undergo motion according to an h-transformed version of the NRW. We aim to write more about this in forthcoming work.

A further difficulty that arises in the use of h-transformed motion for the estimation of λ_* is that the estimates can be dominated by large-deviation effects. Specifically, although the estimates remain unbiased, for large t a substantial contribution to the expectation in (7.11) comes from rare particles with large weights (the exponential, product and indicator terms

Importance sampling:

Relies on choosing new sampling dynamics (previous quote: h function).

Rare-event simulation: Large deviation asymptotics guides the choice.

Ex: Estimate the probability of exiting A for a process $\{X^n(t); t \in [0,T]\}$:

$$p_n = P(X^n(\tau) \in \partial A, \text{ some } \tau \leq T) .$$

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(Standard) Monte Carlo: Sample trajectories $\{X_i^n\}_{i=1}^N$ from original dynamics P^n. Estimator

$$\hat{p}_{N,n} = \frac{1}{N} \sum_{i=1}^{N} I\{X_i^n(\tau) \in A, \text{ some } \tau \leq T\} .$$

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Importance sampling: Sample trajectories $\{X_i^n\}_{i=1}^N$ from alternative dynamics Q^n. Estimator

$$\tilde{p}_{N,n} = \frac{1}{N}\sum_{i=1}^{N} I\{X_i^n(\tau) \in A, \text{ some } \tau \le T\}\frac{dP^n}{dQ^n}(X_i^n).$$

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Choice of Q^n main challenge for IS. Two common approaches:

- Lyapunov inequalities
- Control approach/HJ equations

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A large deviation-style analysis of associated control problem suggests Q^n according to $\nabla_x W$, W a <u>subsolution of a HJ equation:</u>

$$\begin{cases} W_t(t,x) - H(x, -\nabla_x W(t,x)) \geq 0, (t,x) \in (0,T] \times A, \\ W(t,x) \leq 0, \qquad (t,x) \in (0,T] \times \partial A. \end{cases}$$

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In general: Very difficult to construct "optimal" subsolution. So far, mostly toy-ish problems studied in full detail.

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An in-depth (LD-style) analysis for the NTE setting (very) interesting.

- Control-perspective on the choice of h,
- Provably efficient methods for interacting particle systems,
- Combination with on-line estimates and updates,
- Measure-valued processes, different from standard setting.

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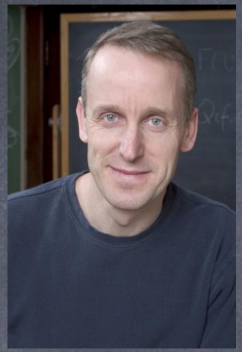
(similar ideas can be applied for MCMC, splitting, genealogical methods...)

Possible alternative: Efficient numerical methods for QSDs


Recent work studies connections between QSDs and stochastic control:

Budhiraja, Dupuis, N., Wu - Quasi-stationary distributions and ergodic control problems (arXiv:2103.00280)

- Introduce ergodic stochastic controls problems associated with the QSD of a diffusion process.
- Prove well-posedness of HJB eq:s and describe how they characterise important properties of the QSD.
- Opens up for numerical methods, from the control community, for approximating the rate (eigenvalue) and the QSD.

(very early in the development, much to do for this to be viable...)

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