ItoVsStratonovich

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Setting up iPython

```
In [1]: import numpy as np
        import matplotlib
        import matplotlib.pyplot as plt
        from pylab import *
```

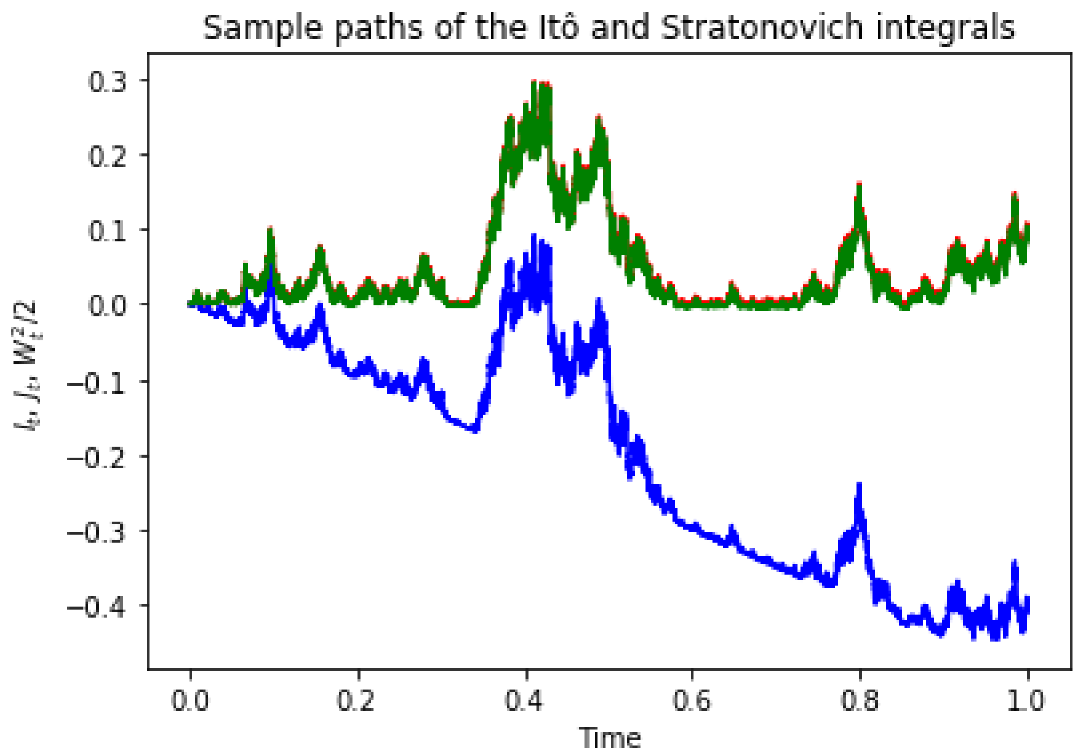
Create a sample Brownian path using i.i.d. normal noise.

```
In [2]: #Normal Increments
        noiseT = lambda N,T: np.random.normal(0,sqrt(T/float(N)),N)

        # Number of time steps:
        N = 100000
        t = np.linspace(0,1,N+1) # time discretisation
        # append a 0 to the start of the noise vector, and compute the cumulative sum:
        W = cumsum(np.hstack((arange(1),noiseT(N,1))))
```

Functions to compute the Itô and Stratonovich integrals for $\int_0^1 W_s \, \mathrm{d}W_s$.

```
In [3]: def Ito(W):
                "This computes the Ito integral of W against itself"
                M = W.size
                I2 = W[0:(M-1)]*np.diff(W[0:(M+1)])
                return cumsum(I2)

        # NB: to get the value at the midpoint, we can only compute
        #    the difference between 0,2,4,...
        def Strat(W):
                "This computes the Stratonovich integral of W against itself"
                M = W.size
                J2 = W[1:(M+1):2]*np.diff(W[0:(M+1):2])
                return cumsum(J2)

        # To make the comparison at equivalent discretisations, we only
        #    consider W at alternate points for the Ito integral
        I = Ito(W[0:size(W):2])
        J = Strat(W)
```

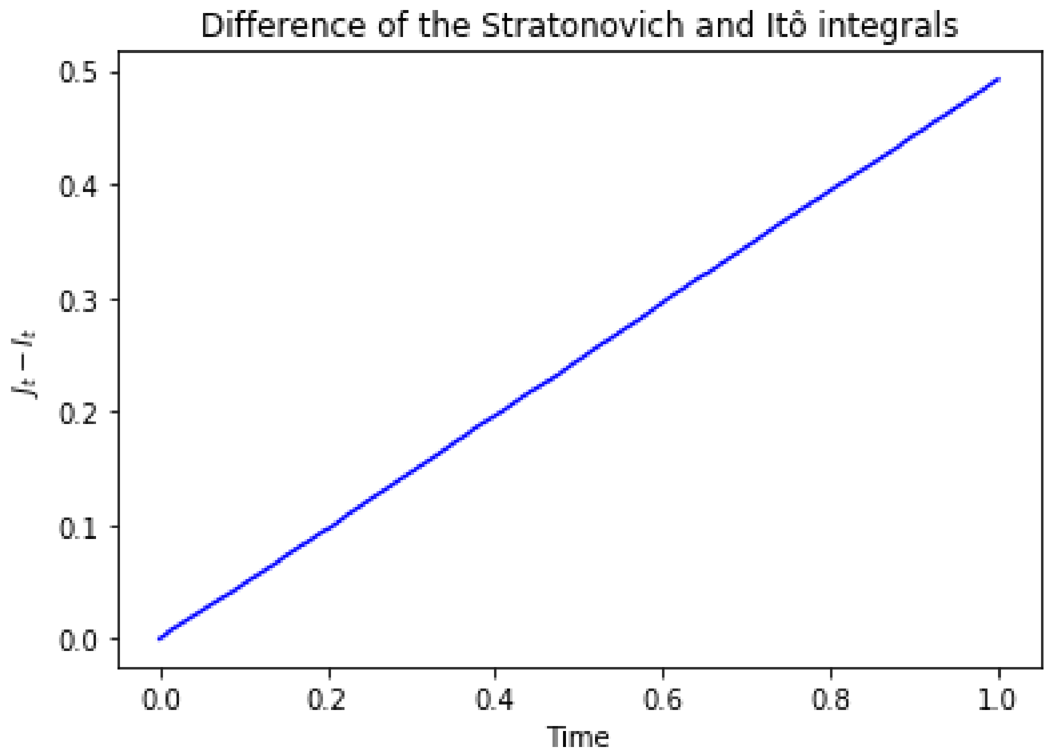
In [4]:
```
%matplotlib inline

fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

axes.plot(t, W*W/2, 'r')
axes.plot(t[0:(size(t)-2):2], I, 'b')
axes.plot(t[0:(size(t)-2):2], J, 'g')

axes.set_xlabel(r'Time')
axes.set_ylabel(r'$I_t$, $J_t$, $W_t^2/2$')
axes.set_title(u'Sample paths of the Itô and Stratonovich integrals');
```



Note that the red curve of $\frac{W_t^2}{2}$ is barely visible behind the green curve, which is the Stratonovich integral.

In [5]:
```
%matplotlib inline

fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

axes.plot(t[0:(size(t)-2):2], J-I, 'b')
axes.set_xlabel(r'Time')
```

```
axes.set_ylabel(r'$J_t - I_t$')
axes.set_title(u'Difference of the Stratonovich and Itô integrals');
```



As might have been guessed, the difference between the Itô and Stratonovich integrals is $t/2$. This is slightly different to the result in lectures: in the lectures, we showed that taking the right-hand endpoint of the interval gives a difference of t between this integral and the Itô integrals. Numerically, this confirms the intuition that taking the mid-point is 'inbeetween' these two cases.