SAMBa Reading List

This list summarises content that is important to the main areas of mathematics within the 'Statistical Applied Mathematics' remit. All students should consider the level of preparation and prior exposure they currently have in these areas before entering SAMBa. In your first year, you will be required to take units in each of the topic areas 'Statistics and Data Science', 'Applied and Probabilistic Analysis and Modelling', and 'Computation and Numerical Mathematics'. The exact units may depend on your existing knowledge, but you will need a basic understanding of each of these areas, and it is usually expected that you have a deeper level of understanding in at least some of these areas.

There are no compulsory taught units, but you are expected to take at least 4 'core' units. Information specific to some of these are given below. The core units also often have content which connects different mathematical areas.

The list below gives suggested texts that should help you refresh or consolidate your background in each of the main areas covered in SAMBa. Complete knowledge of the texts suggested below is not expected, and typically students will be stronger in some areas than others, however in each stream, having a good background understanding will prepare you well for the fast-moving SAMBa first year. You may also find these texts useful as reference during the first year. They are all available from the SAMBa office – if you want to read a text before arrival, e-mail us (samba@bath.ac.uk) and you may be able to borrow the book over the summer.

Computation and Numerical Mathematics

Core units: Scientific Computing; Numerical Linear Algebra; Numerical Solution of Elliptic PDEs

Most SAMBa students will have taken introductory courses on Numerical Analysis before arrival. Numerical mathematics in Bath typically uses Matlab for computation, and a brief introduction to Matlab will be given in induction week. SAMBa students are expected to quickly become familiar with the following topics:

- Basic MATLAB Programming for Numerical Analysis
- Floating point numbers and rounding error

- Concepts of Convergence and Accuracy: Order of convergence, extrapolation and error estimation
- Approximation of Functions: Polynomial interpolation, error analysis
- Integration: Newton-Cotes formulae. Gauss quadrature. Composite formulae. Error analysis
- ODEs: Euler, Backward Euler, Trapezoidal and explicit Runge-Kutta methods. Stability and convergence
- Linear Algebra: Gaussian elimination, LU decomposition, pivoting, Matrix norms, conditioning, backward error analysis, iterative refinement, regularization of ill-posed linear systems

The list below contains good references for this introductory material:

- Moler, Numerical Computing with Matlab
- Atkinson, An Introduction to Numerical Analysis, Wiley
- Süli and Mayers, An Introduction to Numerical Analysis, Cambridge University Press
- Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge University Press
- Hansen, Discrete Inverse Problems, SIAM

The above books cover more than is needed. In addition, below are the references which are used during the core SAMBa unit *Numerical Linear Algebra*:

- Golub and Van Loan, Matrix Computations, Johns Hopkins University Press.
- Demmel, Applied Numerical Linear Algebra, SIAM.

Statistics and Data Science

Core units: Applied Statistical Inference; Bayesian and Large Scale Methods; Machine Learning

Before entering SAMBa, students should be familiar with the following topics in statistics:

- Hypothesis testing
- Properties of multivariate normal random variables
- The central limit theorem
- Linear models

Some knowledge of maximum likelihood estimation and generalised linear models would also be beneficial.

The SAMBa core unit *Applied Statistical Inference* is based on the textbook Core Statistics, which is available online: <u>Core Statistics by Simon Wood</u>. Most statistical programming is done in the R programming language, and some statistical programming will also be done in Python, particularly for Machine Learning. A brief introduction to R and Python will also be given in induction week.

Books covering relevant material include the following:

- Dalgaard, Introductory Statistics with R, Springer Science & Business Media
- Faraway, Linear Models with R, CRC Press
- Faraway, Extending the Linear Models with R, CRC Press

For students with a strong interest in statistics, the following books contain material that is useful for Applied Statistical Inference, and a great deal more besides.

- Davison, Statistical Models, Cambridge University Press
- Gelman et al., Bayesian Data Analysis, Chapman & Hall/CRC
- Held and Bove Applied Statistical Inference, Springer

Students who find this material challenging may find helpful more introductory texts such as:

• Ross, Introductory Statistics, Elsevier

Applied and Probabilistic Analysis and Modelling

Core units: Inverse Problems, Data Assimilation and Filtering; Advanced Numerical Computation; Applied Stochastic Differential Equations
This area encompasses a wider range of academic content and stretches further across different traditional disciplines. Depending on your background and interests, you may specialise in some areas and only lightly touch on other areas.

Differential Equations and Dynamical Systems

Includes: systems of linear ODEs, eigenvectors, fundamental matrices and matrix exponentials, exponential stability, Routh-Hurwitz criterion, solution of inhomogeneous systems by variation of parameters, distributions, Laplace transforms, equilibria, stability, non-linear systems, chaos, codimension-one bifurcations (saddle-node, Hopf) of equilibria, periodic orbits, Poincaré

maps, bifurcations of periodic orbits, global bifurcations, Lyapunov exponents and chaos.

Reading material includes:

- Bender and Orszag, Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory, Springer
- Glendinning, Stability, Instability and Chaos, Cambridge University Press

Probability

Includes: basic combinatorics, independence, random variables, discrete and continuous probability distributions, multivariate Normal distribution, law of large numbers, central limit theorem, conditional probability, conditional expectation, Markov processes, basic measure theory.

Reading material includes:

- Ross, A First Course in Probability (9th ed.), Pearson
- Grimmet and Stirzaker, Probability and Random Processes (3rd ed.),
 Oxford. (Relevant Chapters: 1 and 6.1-6.11)
- Williams, Probability with Martingales, Cambridge University Press.
 (For an introduction to measure-theoretic probability. Relevant Chapters: 0-6)
- Bartle, The Elements of Integration and Lebesgue Measure, Wiley (For those with no measure-theoretical background. Relevant Chapters: 1-6.)

Those interested in taking the core unit *Applied Stochastic Differential Equations* should ensure that they read and are familiar with the following prerequisite material in probability: <u>MA50251 Prerequisite Material</u>.

Partial Differential Equations

Includes: solution methods for Laplace's equation, the heat equation and the wave equation in simple geometries in one, two and three space dimensions. Dirichlet and Neumann boundary conditions. Green's functions. Fourier series solutions. Fourier and Laplace Transforms. Reading material includes:

- Pinchover and Rubinstein, An Introduction to Partial Differential Equations, Cambridge University Press
- Strauss, Partial Differential Equations: An Introduction, Wiley

Mathematical Biology

Includes: population dynamics of one or more species in discrete and continuous time, infectious diseases, population genetics, biological motion, molecular and cellular biology, pattern formation in biological systems, and tumour modelling.

Reading material includes:

■ Britton, Essential Mathematical Biology, Springer

Fluid Mechanics and Geophysical Flows

Includes: streamlines and particle paths. Euler and Navier-Stokes equation for incompressible flow. Reynolds number. Streamfunctions. Vorticity. Invisicid flows and Bernoulli's equation. Potential flow (e.g. around a cylinder) and Stokes flow. Boundary layers.

Reading material includes:

Acheson, An introduction to fluid dynamics, Oxford University Press