One-Dimensional Model for Faraday Wave-Droplet Dynamics

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Set Up

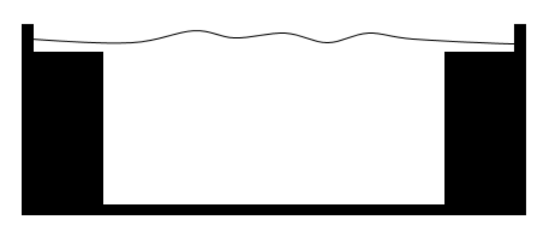


Figure 1: Bath of Vibrating Fluid

Dimensionless Forcing Acceleration

$$\Gamma = \frac{A\omega_0^2}{g}$$

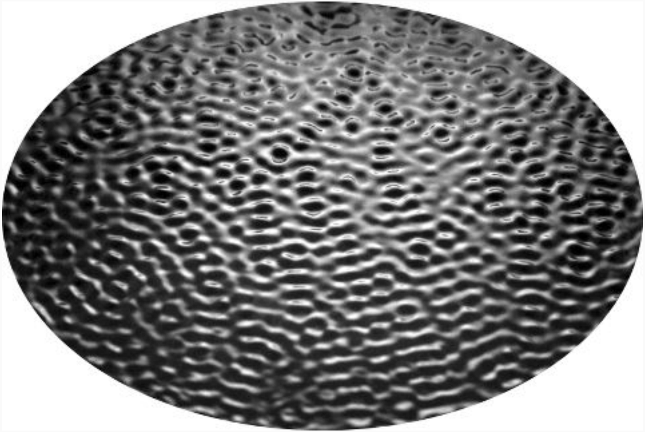


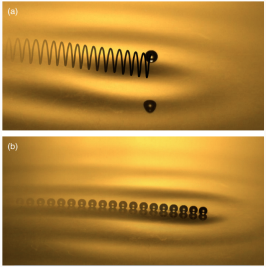
Figure 2: Faraday Waves

Faraday Threshold

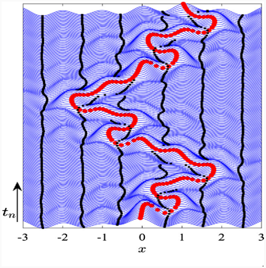
$$\Gamma = \Gamma_F$$

Subcritical Phenomena

Periodic Bouncer



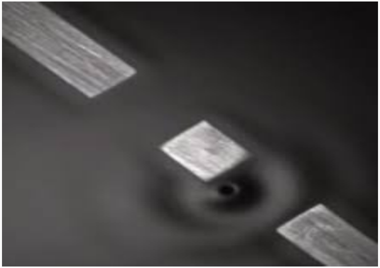
Walker



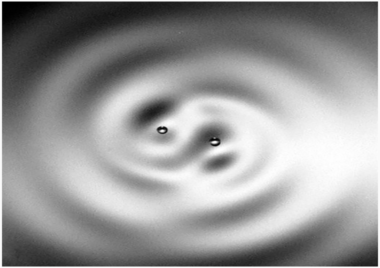
Chaos

Quantum Analogies

- Diffraction
- Interference
- Quantised Orbits
- Tunneling
- Wave-like Statistics



Double Slit Experiment



Quanitised Orbiting Droplets

A Discrete Time One-Dimensional Model

- Each time step correlates with droplet's impact on the surface
- On Impact, wavefield and droplet are subjected to jump conditions
- Method aids us in preforming simulations

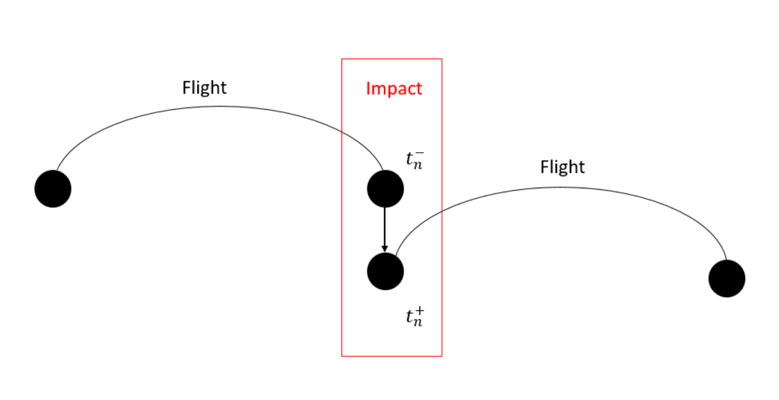


Figure 3: Discrete Time Model

System of Equations

$$\eta(x,t) = \sum_{k=k_1}^{k_K} a(k,t)\cos(kx) + b(k,t)\sin(kx)$$

$$X_{N+1} = X_N + \dot{X}_N$$

$$\mathbf{u}_{N+1} = M(k,\Gamma)\mathbf{u}_N + \mathbf{J}(X)$$

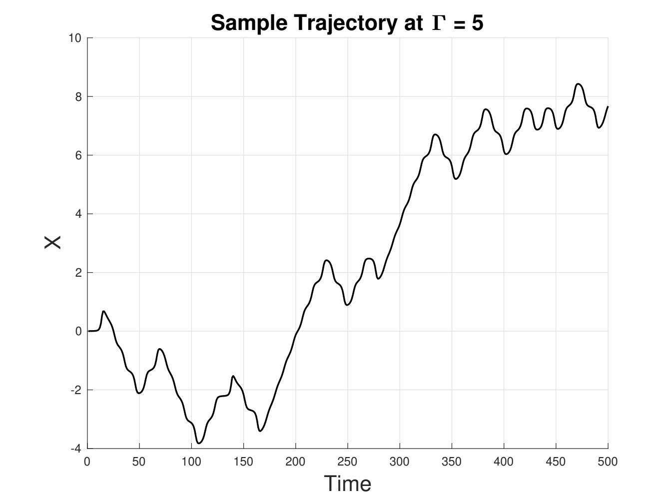
$$\dot{X}_{N+1} = \dot{X}_N + F(\eta_x(X_{N+1}, t = N+1))$$

Steady State Analysis

- Bouncer (stable until Γ_W)
- Walker (stable between Γ_W & Γ_C)
- Periodic Orbiter (highly unstable)

Steady State Analysis

In the Chaotic regime, the droplet's motion alternates between a periodic orbiter and a walker.



Steady State Analysis - Periodic Orbiters

Discrete-time periodic orbiter correlates with specific Γ values.

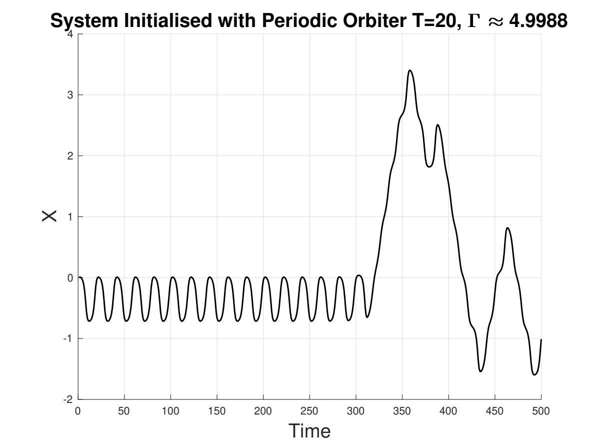


Figure 5: Periodic Orbiter is highly unstable

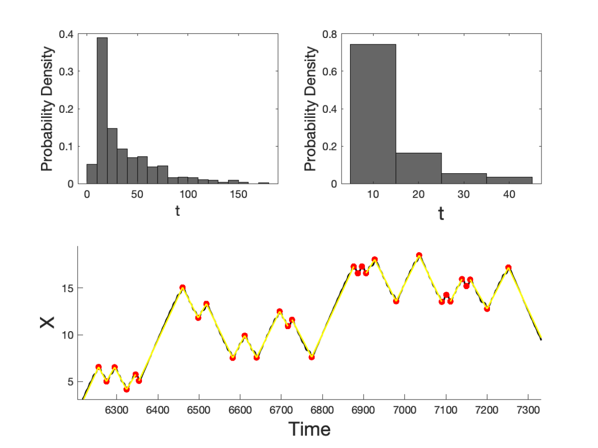
Steady State Analysis - Periodic Orbiters

T	Γ	Memory
19	5.0538	44.6
20	4.9988	26.18
21	4.9490	18.51
22	4.9055	14.99
23	4.8693	12.59
24	4.8385	11.24
25	4.8123	10.15
26	4.7905	9.527
27	4.7725	8.979

Memory

$$|\Lambda(\Gamma)|^{\text{Mem}} = \frac{1}{e},$$

A Run & Tumble Model



1. Red dots marked at each turning point
2. Red dot denotes a “tumble” of duration half a period
3. Run connects turning points

Reduced Modelling

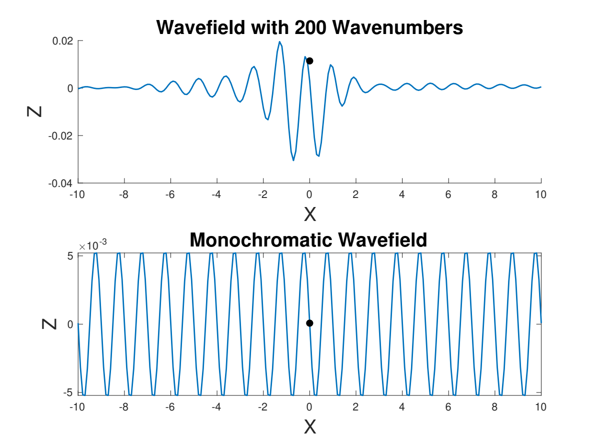
What have we done so far?

1. 1D
2. Discrete Time

What more can we do?

1. A monochromatic Wavefield $k = k_F = 2\pi$
2. Further Reduction by approximating the Floquet Matrix M.

A Monochromatic wavefield



$$\eta(x,t) = a\cos(k_F x) + b\sin(k_F x)$$

Approximation to the Floquet Matrix

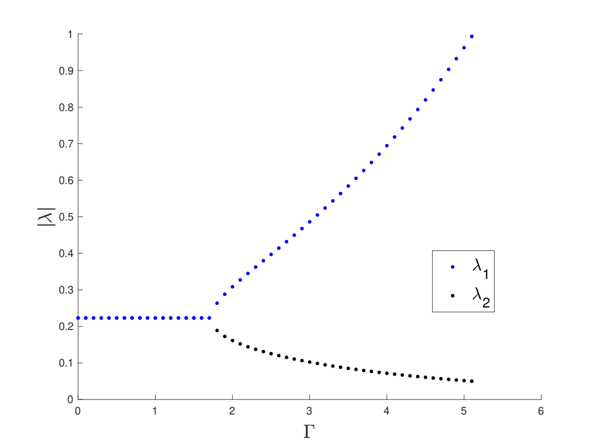


Figure 6: Eigenvalues of the Floquet Matrix M

$$M = \mathbf{w}_1^T \lambda_1 \mathbf{v}_1 + \mathbf{w}_2^T \lambda_2 \mathbf{v}_2$$

From 5 Variables to 3

$$a_{N+1} = -\alpha\lambda_1 \cos(k_F \dot{X}_N) + \lambda_1 \cos(k_F \dot{X}_N) a_N + \lambda_1 \sin(k_F \dot{X}_N) b_N, \tag{1}$$

$$b_{N+1} = \alpha\lambda_1 \sin(k_F \dot{X}_N) + \lambda_1 \cos(k_F \dot{X}_N) b_N - \lambda_1 \sin(k_F \dot{X}_N) a_N, \tag{2}$$

$$\dot{X}_{N+1} = \dot{X}_N - \mu \left(\varphi k_F b_{N+1} + \dot{X}_N \right). \tag{3}$$

Bifurcation Curves

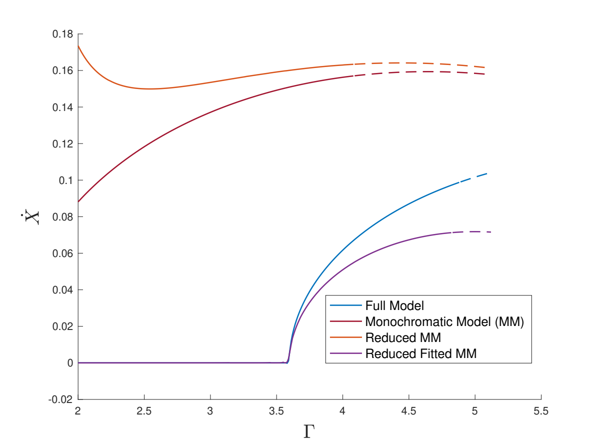


Figure 7: Bifurcation Curve of the Reduced Models

Periodic Orbiters in the Reduced Monochromatic Model

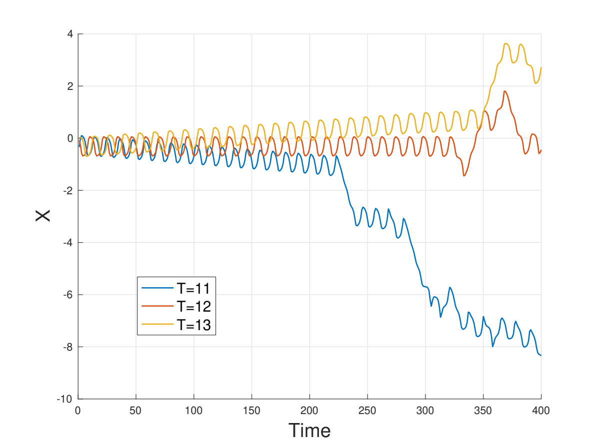


Figure 8: Periodic Orbiters

Drift in Periodic Orbiters

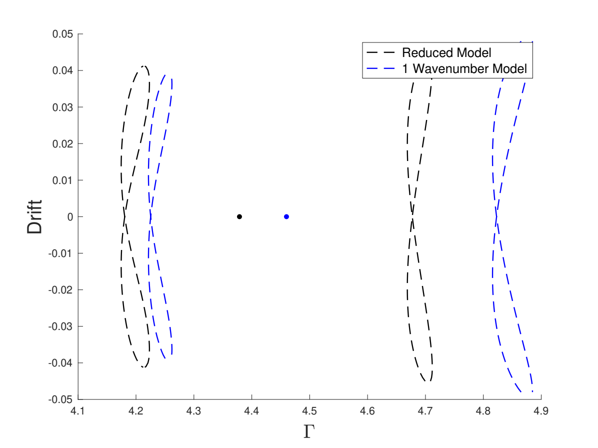


Figure 9: Many orbiters exists with different drifts

Thank you

Any Questions ?