Weakly quenched scaling limit for the one-dimensional, biased random walk on heavy-tailed conductances

Student: Carlo Scali
Supervisor: Daniel Kious

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University of Bath

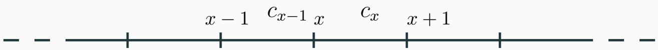
Introduction

The biased RWRC (1)

Bond randomness (RWRC)

Define a family $\{c_x = c(\{x, x+1\})\}_{x\in\mathbb{Z}}$ of i.i.d. positive random variables associated with the edges of $\mathbb{Z}$, then deterministically tilt them and denote: $c_x^\lambda = e^{\lambda x} c_x$ for $\lambda > 0$. It is natural to define a family of jump probabilities

$$\omega_x := \frac{c_x^\lambda}{c_{x-1}^\lambda + c_x^\lambda}.$$



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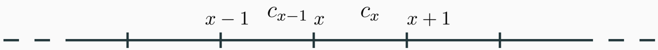

Consider the Markov chain with those jump probabilities

$$P^\omega \left[X_{t+1} = y | X_t = x\right] = \begin{cases} \omega_x & y = x+1, \\ 1 - \omega_x & y = x-1, \\ 0 & \text{otherwise.} \end{cases}$$

The biased RWRC (2)

Remark

There is an “old” (Tavaré, Zeitouni 2004) theorem that guarantees that, for almost every realisation of the environment, the random walk is such that

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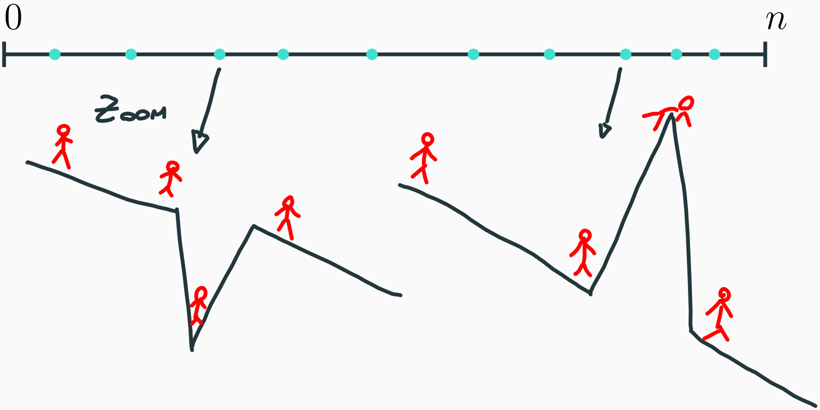
Hypothesis on the conductances

We assume that, for $\alpha \in (0, 1)$

$$\mathbb{P}[c_0^{-1} > t] \sim \text{const.}\, t^{-\alpha} \quad \text{or} \quad \mathbb{P}[c_0 > t] \sim \text{const.}\, t^{-\alpha}.$$

Heuristics



Previous result and a question

Annealed law

The **annealed** law $\mathbb{P} \otimes P^\omega[\cdot]$ can be attained by averaging out the randomness of the environment, that is

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Theorem (Berger, Salvi 2020)

Under $\mathbb{P} \otimes P^\omega[\cdot]$

$$\frac{T_n}{n^{1/\alpha}} \to \text{const.} \times S_\alpha.$$

Where S_α is some known (α-stable) distribution.

Some results

Results (1)

Theorem

There exists a random subsequence of the space x_m, depending just on the environment, such that $\mathbb{P}$-a.s.

$$\lim_{m\to\infty} P^\omega\left[\frac{T_{x_m}-E^\omega\left[T_{x_m}\right]}{\sqrt{\mathrm{Var}^\omega(T_{x_m})}}\leqslant y\right]=\Phi(y).$$

Where $\Phi(\cdot)$ is the c.d.f. of a standard Gaussian.

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Where $\Psi(\cdot)$ is the c.d.f. of a standard Exponential random variable.

Results (2)

Thanks to the two previous theorems we have ruled out the possibility of a fully quenched result, can we still do a bit better than the annealed limit?

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Theorem

In the metric space of probability measures on $\mathbb{R}$ with finite first moment, endowed with a proper distance

$$\mathscr{L}\left(\frac{T_n}{n^{1/\alpha}}\middle|\omega\right) \stackrel{\text{dist}}{\to} \mathscr{L}\left(\sum_{p\geqslant 1} \widehat{\omega}_p e_p \middle| (\widehat{\omega}_p)_{p\geqslant 1}\right) \qquad \text{in } \mathbb{P}\text{-law},$$

where $\widehat{\omega} := \{\widehat{\omega}_p\}_p$ is a Poisson point process (p.p.p.) with intensity $\alpha\,\mathrm{const.}\,\widehat{\omega}^{-(1+\alpha)}d\widehat{\omega}$, $\{e_p\}_p$ are i.i.d. exponential random variables with parameter 1.

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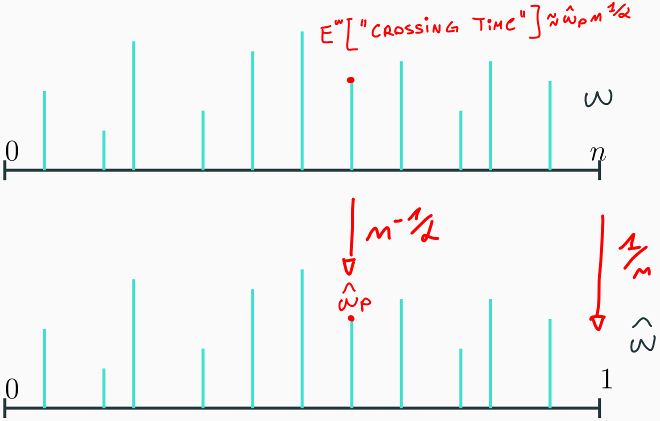

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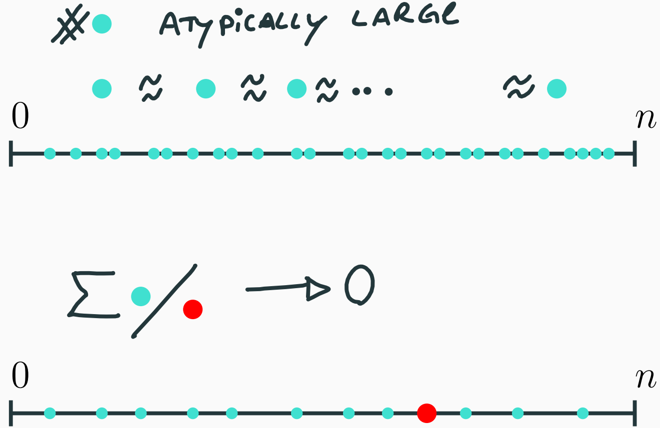

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- Proved that a quenched limit is impossible.
- Characterised two very different sub-sequential regimes that make the quenched limit false.
- Proved a second-best “weakly-quenched” result that precisely characterises the process.

Next (current) steps:

- Without bias?
- Dimension $d \geqslant 2$?

Questions?

Thank You!