Two Species Contact Processes

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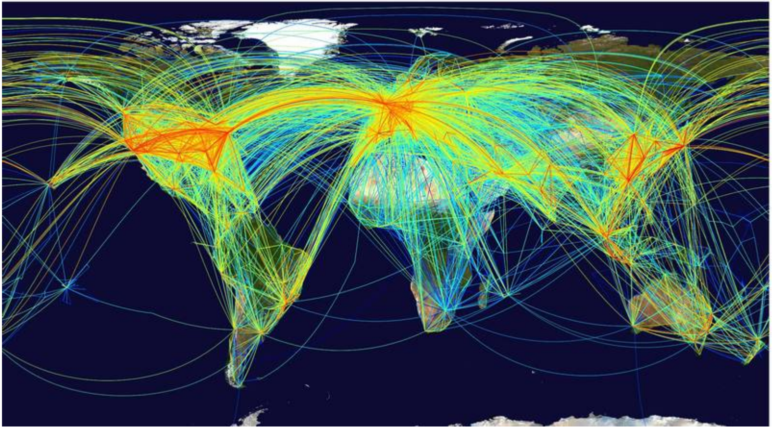
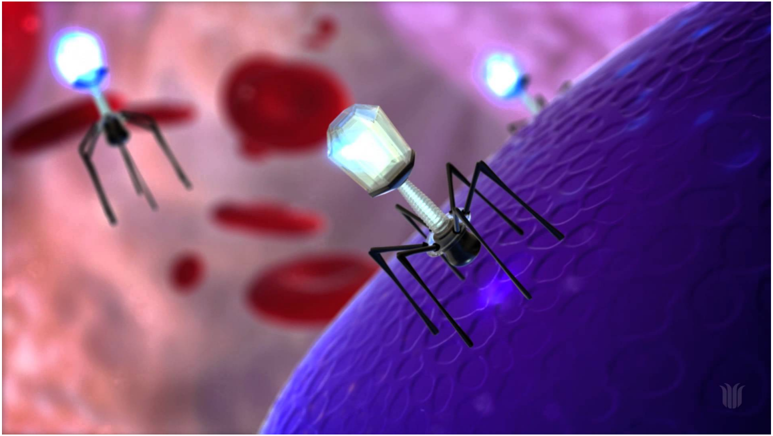
June 19, 2017

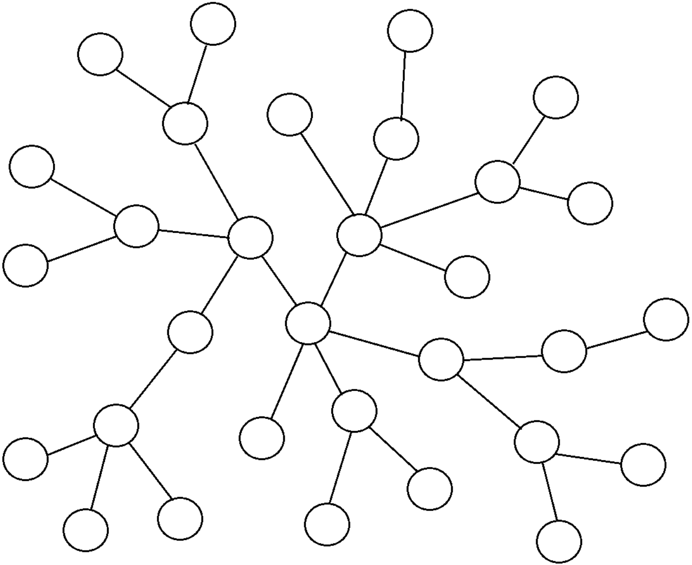
[1] with Tim Rogers, Peter Mörters

Epidemics

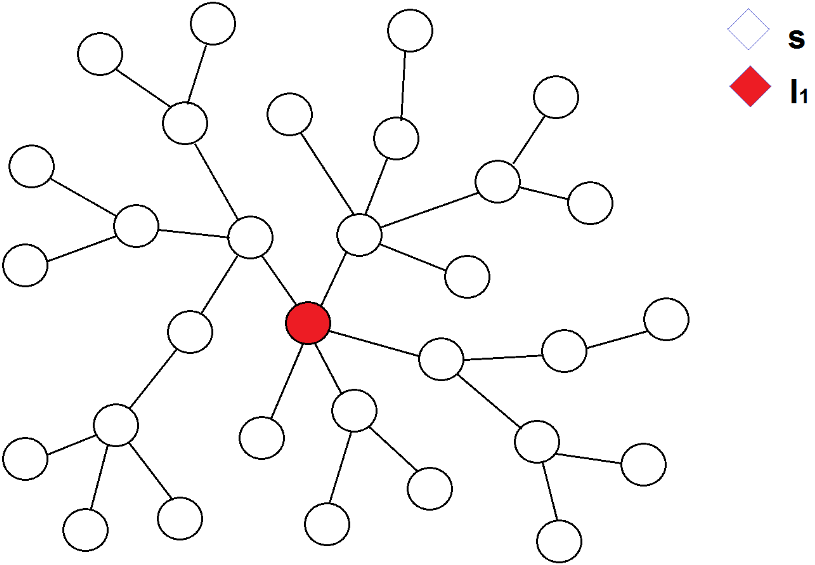


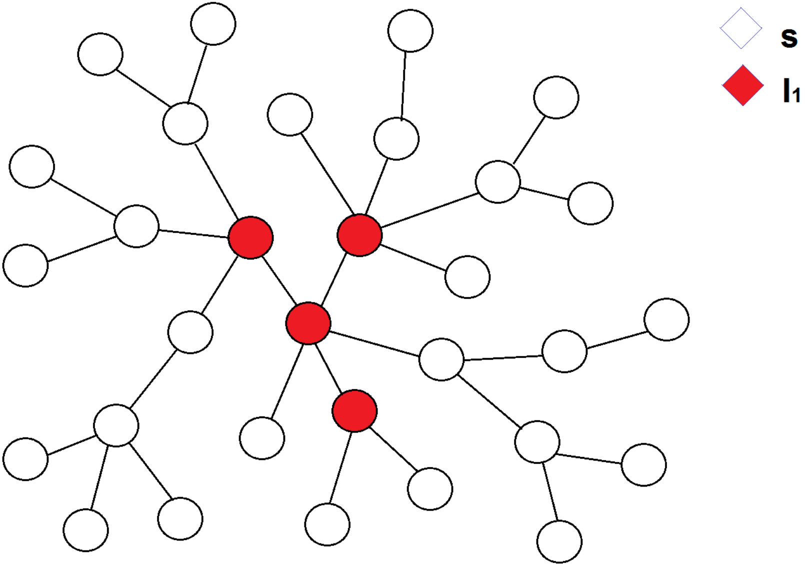
Bacteriophage

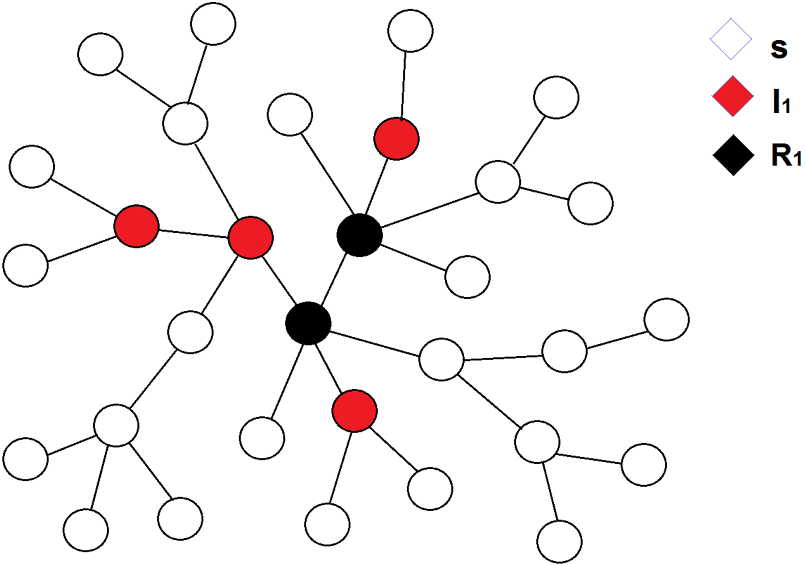


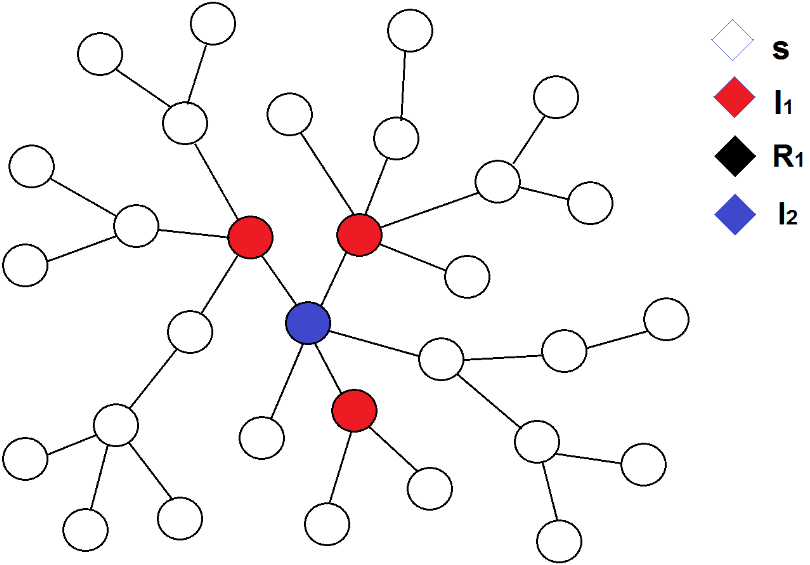


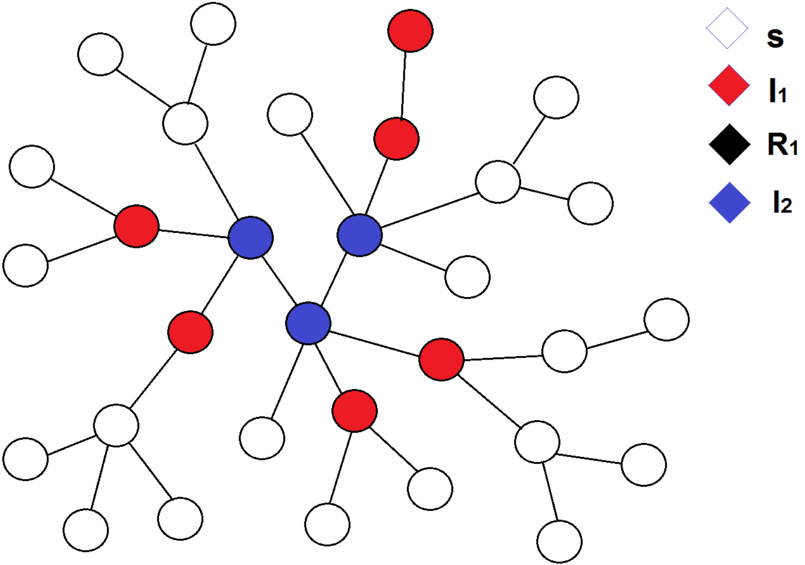
S

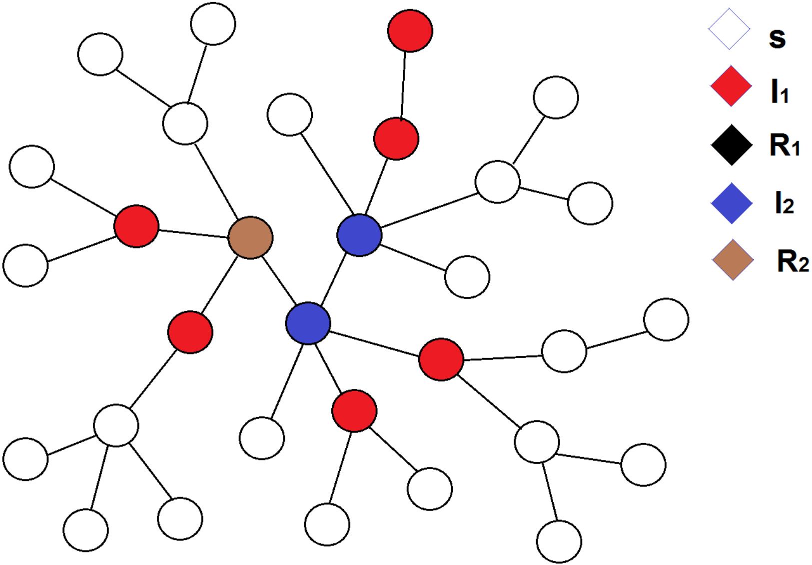

S
I1

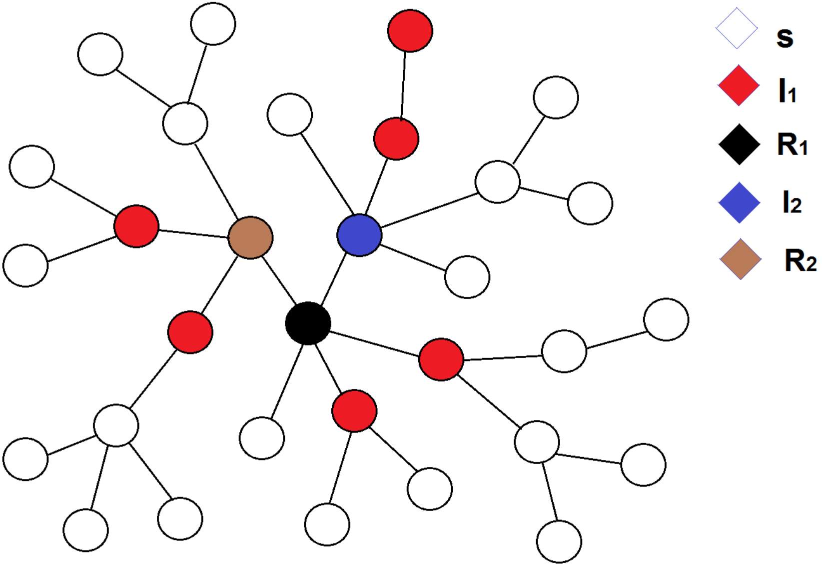

S
I1

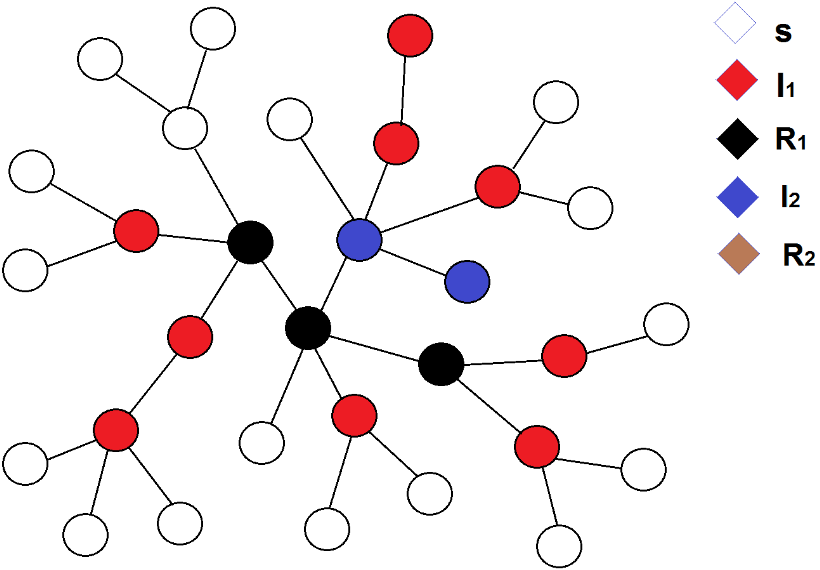

S
I1
R1

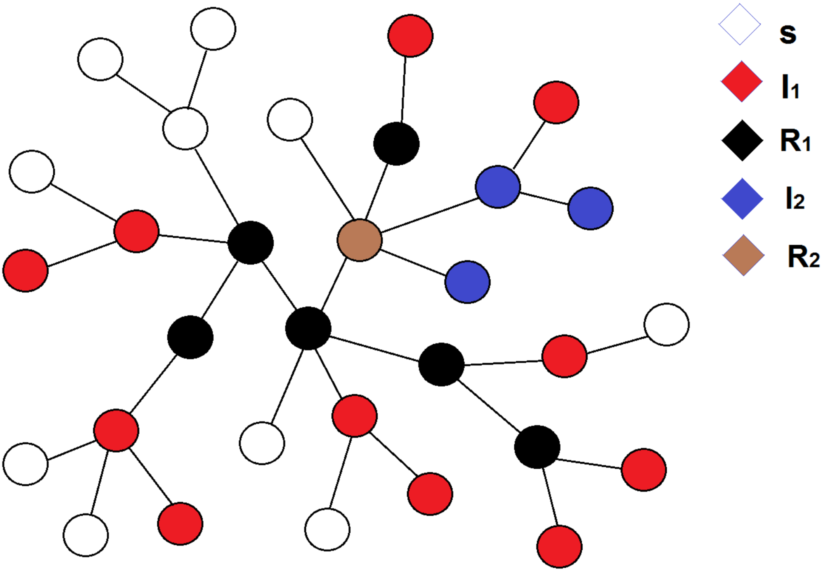

S
I1
R1
I2

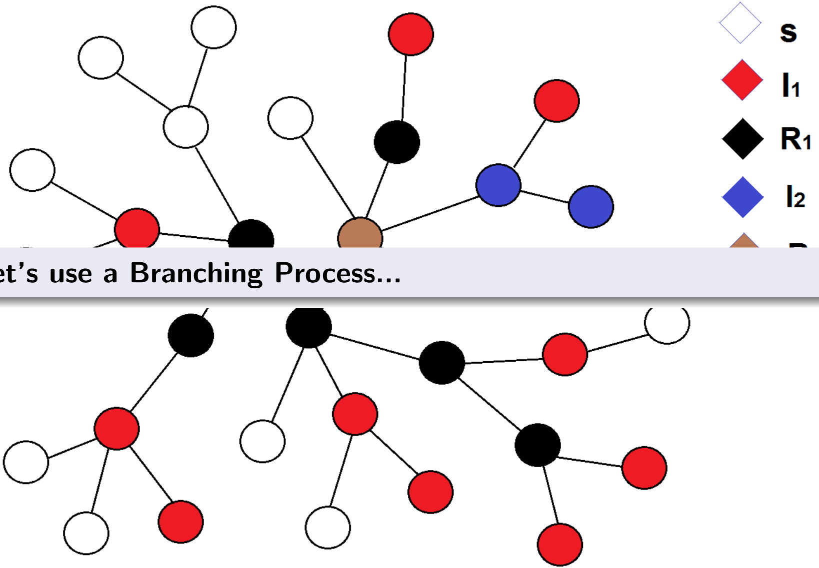

S
I1
R1
I2


S
I1
R1
I2
R2

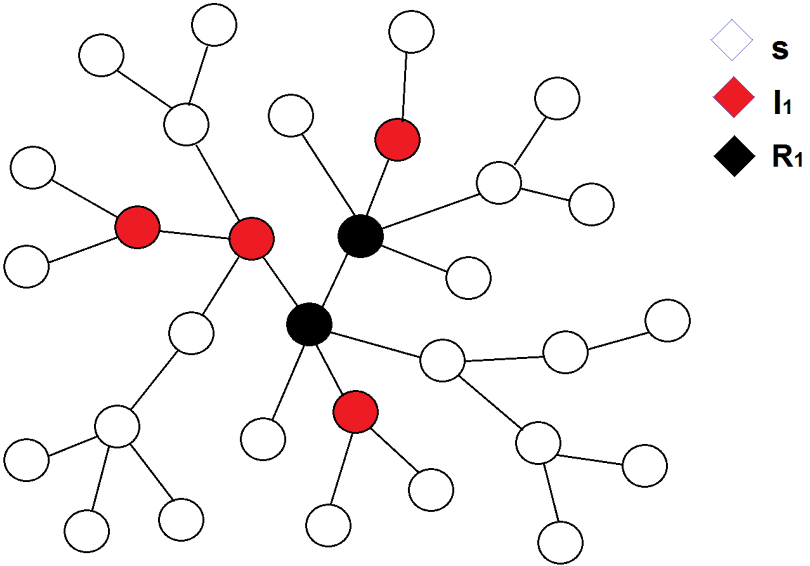

S
I1
R1
I2
R2


S
I1
R1
I2
R2

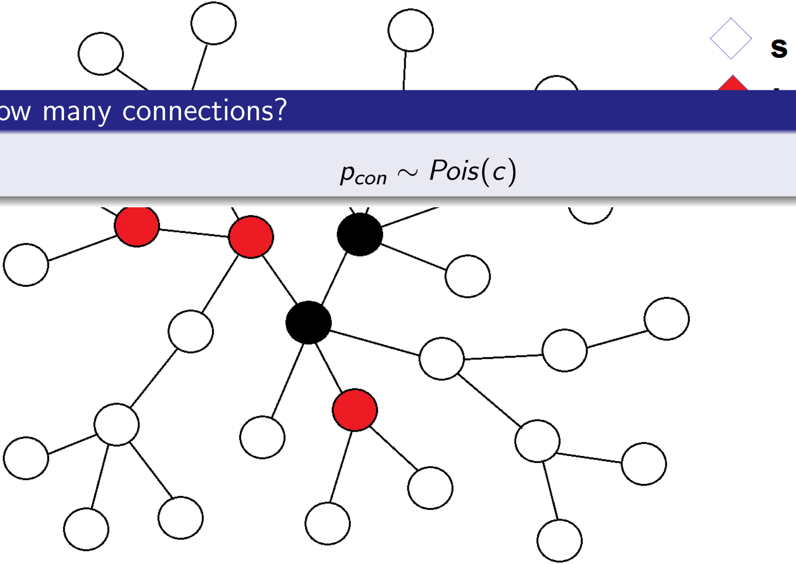

S
I1
R1
I2
R2



Let's use a Branching Process...

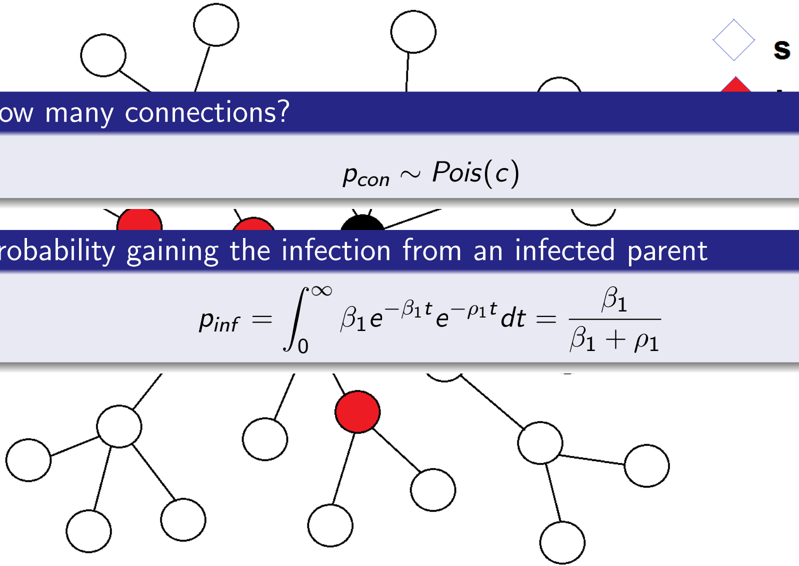


S
I1
R1



How many connections?

$$p_{con} \sim Pois(c)$$

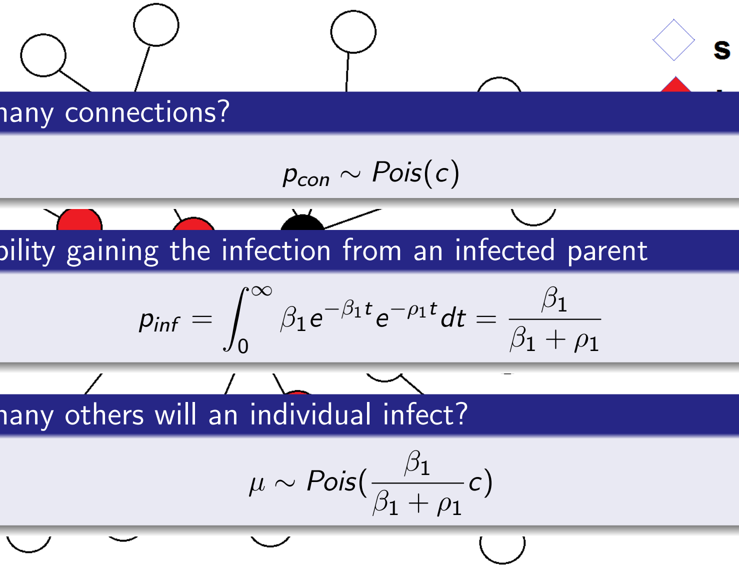


How many connections?

$$p_{con} \sim Pois(c)$$

Probability gaining the infection from an infected parent

$$p_{inf} = \int_0^\infty \beta_1 e^{-\beta_1 t} e^{-\rho_1 t} dt = \frac{\beta_1}{\beta_1 + \rho_1}$$



How many connections?

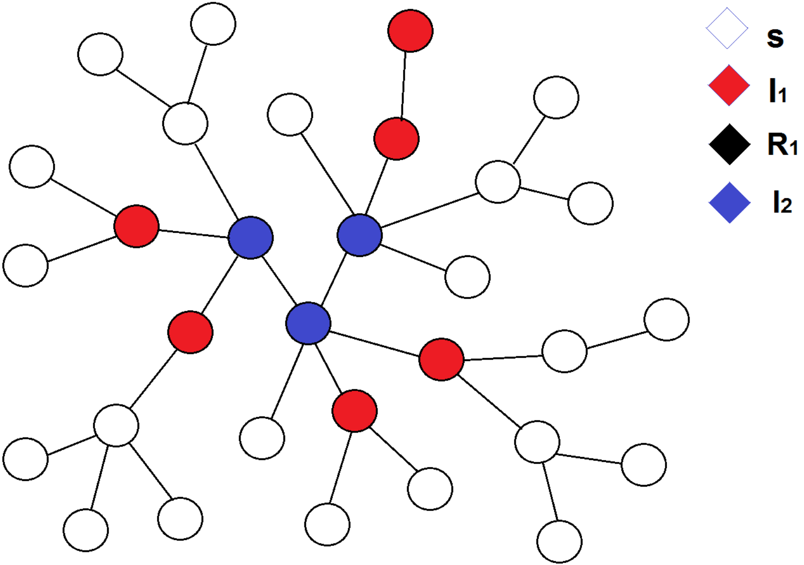
$$p_{con} \sim Pois(c)$$

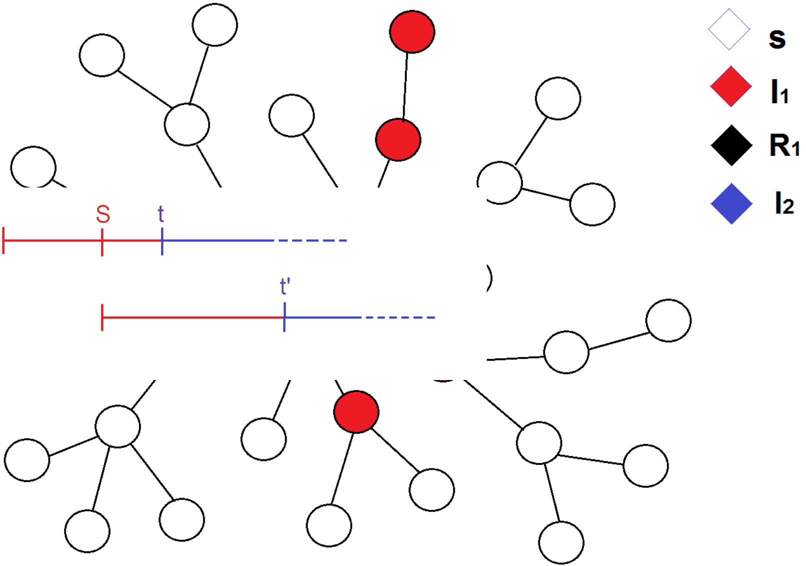
Probability gaining the infection from an infected parent

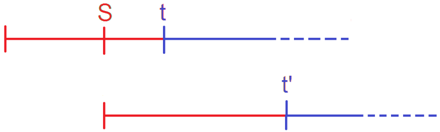
$$p_{inf} = \int_0^\infty \beta_1 e^{-\beta_1 t} e^{-\rho_1 t} dt = \frac{\beta_1}{\beta_1 + \rho_1}$$

How many others will an individual infect?

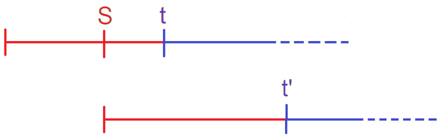
$$\mu \sim Pois(\frac{\beta_1}{\beta_1 + \rho_1} c)$$


S
I1
R1
I2

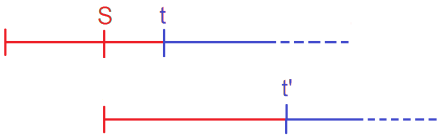

S
t
t'
s
I1
R1
I2



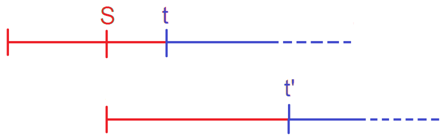
$$\mu(t'|t) =$$



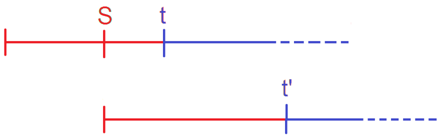
$$\mu(t'|t) = \int_{0\vee(t-t')}^{t}$$



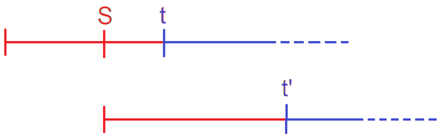
$$\mu(t'|t) = \int_{0 \vee (t-t')}^{t} [\beta_1 e^{-\beta_1 s}]$$



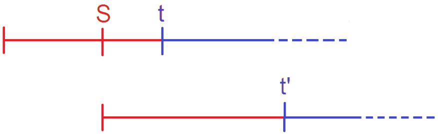
$$\mu(t'|t) = \int_{0\vee(t-t')}^{t} [\beta_1 e^{-\beta_1 s}][\beta_2 e^{-\beta_2(t'-t+s)}]$$



$$\mu(t'|t) = \int_{0\vee(t-t')}^{t} [\beta_1 e^{-\beta_1 s}][\beta_2 e^{-\beta_2(t'-t+s)}][e^{-\rho_1(t-s)}]$$



$$\mu(t'|t) = \int_{0\vee(t-t')}^{t} [\beta_1 e^{-\beta_1 s}][\beta_2 e^{-\beta_2(t'-t+s)}][e^{-\rho_1(t-s)}][e^{-(2\rho_1+\rho_2)(t'-t+s)}]ds$$



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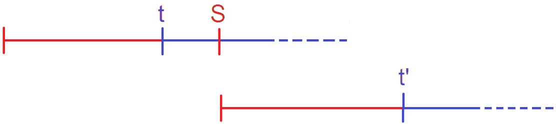
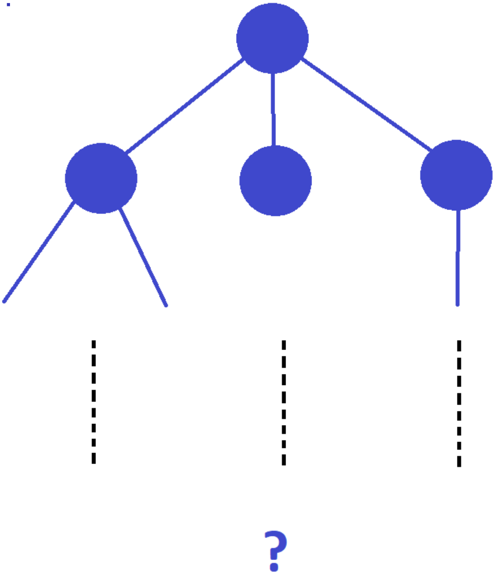


$$\mu(t'|t) = \int_{t}^{\infty} [\beta_1 e^{-\beta_1 s}][\beta_2 e^{-\beta_2 t'}][e^{-(\rho_1+\rho_2)(s-t)}][e^{-(2\rho_1+\rho_2)t'}]ds$$

Endemic?



Endemic?

Distribution of types: $\psi(t)$
Mean number of offspring: m
Population: p

Number of type t

$$mp\psi(t) =$$

?

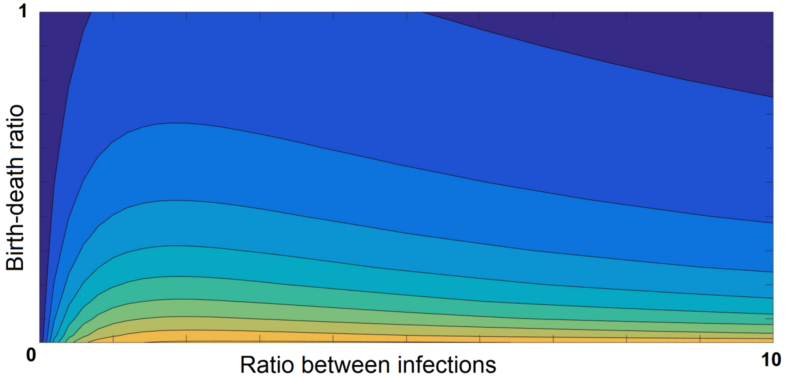
Endemic?

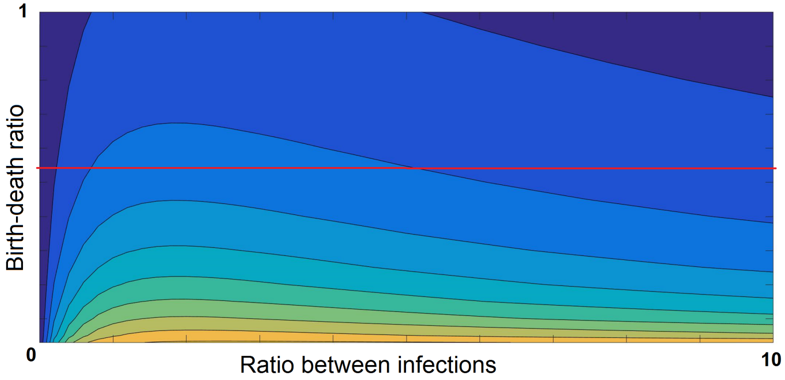
Distribution of types: $\psi(t)$
Mean number of offspring: m
Population: p

Number of type t

$$mp\psi(t) = \int_0^\infty \mu(t|t')cp\psi(t')\,dt'$$

?


1
Birth-death ratio
0
Ratio between infections
10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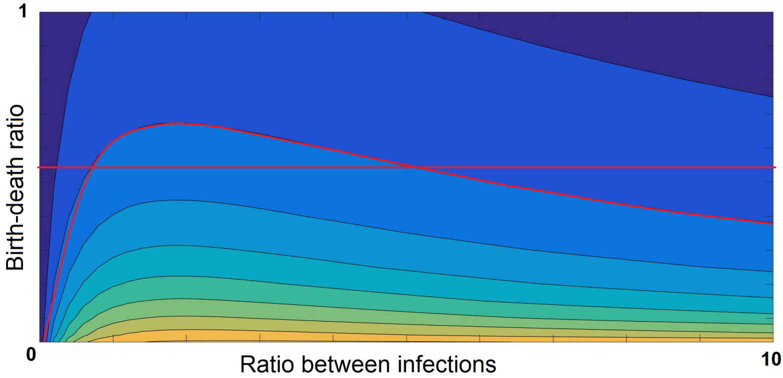
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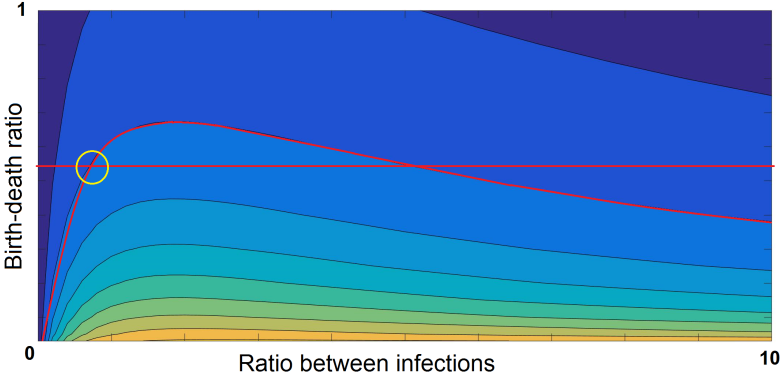
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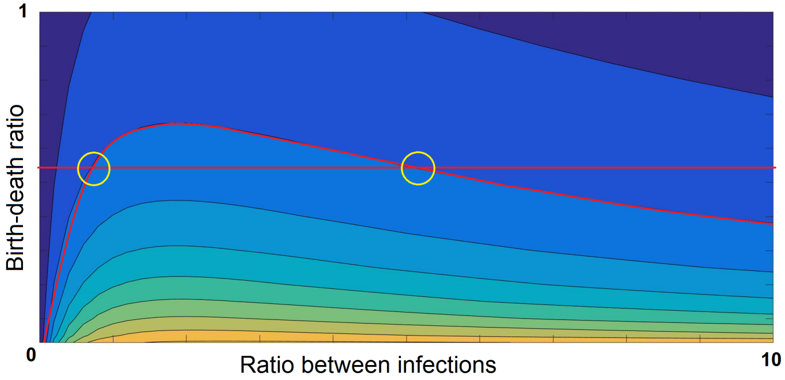
1
Birth-death ratio
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10

Questions?