Macroparasites and Metapopulations

For Muggles

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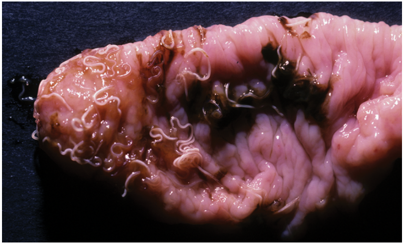
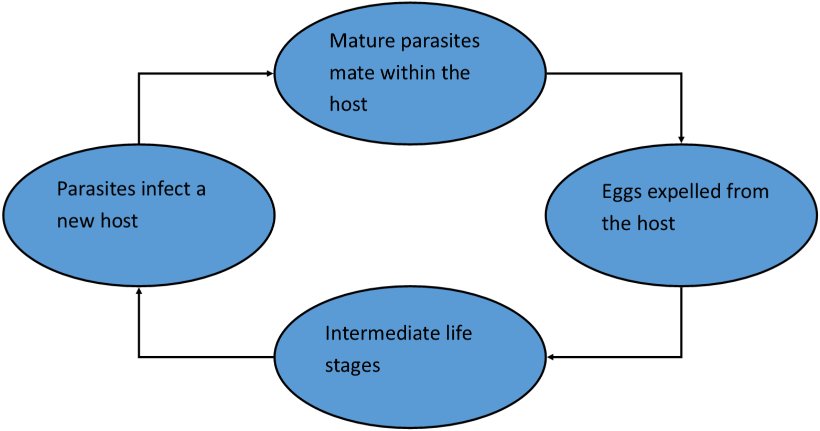
Parasites: Introducing the complex little blighters

- Parasites have more complex lifecycles than standard diseases and as a result aren't often spread through direct contact.
- Parasitic infections also aren't as clear cut as infected or not
- We need to try and formulate models that account for both of these problems



Lifecycle

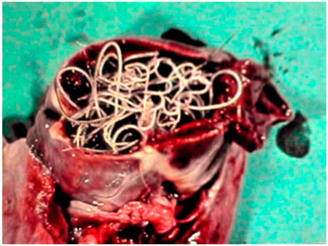

Mature parasites mate within the host
Eggs expelled from the host
Intermediate life stages
Parasites infect a new host

Ode to our forefathers[1,2]

$$\frac{dH}{dt} = (a-b)H - \alpha P.$$

$$\frac{dP}{dt} = \frac{\lambda P(t)H(t)}{H_0 + H(t)} - (b+\mu)P(t) - \alpha H\mathbb{E}[i^2]$$

$$\frac{dP}{dt} = \frac{\lambda PH}{H_0 + H} - (b+\mu+\alpha)P - \frac{\alpha(k+1)P^2}{kH}$$

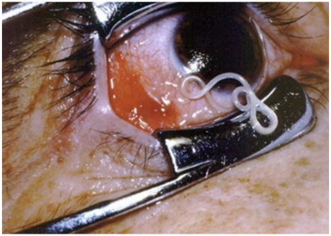


[1] Anderson and May 1978.
[2] May and Anderson 1978.

#So Random[3]

$$\frac{dP_m(t)}{dt} = -(\mu_H(t) + \alpha m + (1 - h_0)\phi + \mu_M m)P_m(t) + \mu_M(m+1)P_{m+1}(t) + \phi \sum_{c=1}^{m} P_{m-c}(t)h_c$$

$$\frac{\partial Q(t;z)}{\partial t} = (\phi(h(z)-1)+\alpha\mathbb{E}(M(t)))Q(t;z)-((\alpha+\mu_M)z-\mu_M)\frac{\partial Q(t;z)}{\partial z}$$



[3] Isham 1995.

Lets get Meta[4]

$$\frac{dL}{dt} = \sum_{i=1}^{N} \beta M_i^2 - \mu_L L - N\phi L^2$$

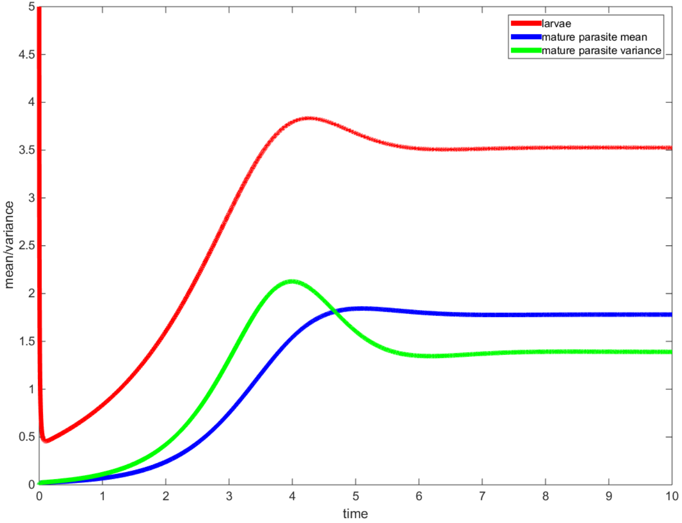
$$\frac{dM_i}{dt} = \rho\phi L^2 - \mu_M M_i - D_M M_i^2$$

[4] [illegible]

A moment like this

$$
\begin{aligned}
\frac{d\overline{L}}{dt} &= N\beta(V_M + \overline{M}) - \mu_L \overline{L} - D_L \overline{L}^2 - N\phi \overline{L}^2 \\
\frac{d\overline{M}}{dt} &= \rho\phi \overline{L}^2 - \mu_M \overline{M} - D_M(V_M + \overline{M}^2) \\
\frac{dV_M}{dt} &= \rho\phi \overline{L}^2(2\overline{M} + 1) - \mu_M(2V_M - \overline{M}) \\
&\quad - D_M(2T_M + 2\overline{M}^3 + 6\overline{M}(V_M) - V_M - \overline{M}^2)
\end{aligned}
$$

What does a numerical Dalek say? - Approximate!



Vive la resistance

$$\frac{dM^i_{ss}}{dt} = \rho\phi L_{ss}(L_{ss} + L_{sr} + L_{rr}) - \mu_M M^i_{ss} - D_M M^i_{ss}((M^i_{ss} + M^i_{sr} + M^i_{rr}))$$

$$\frac{dM^i_{rs}}{dt} = \rho\phi L_{sr}(L_{ss} + L_{sr} + L_{rr}) - \mu_M M^i_{sr} - D_M M^i_{ss}((M^i_{ss} + M^i_{sr} + M^i_{rr}))$$

$$\frac{dM^i_{rr}}{dt} = \rho\phi L_{rr}(L_{ss} + L_{sr} + L_{rr}) - \mu_M M^i_{ss} - D_M M^i_{rr}((M^i_{ss} + M^i_{sr} + M^i_{rr}))$$

and the beat goes on...

$$\frac{dL_{ss}}{dt} = -(\mu_L + D_L(L_{ss} + L_{sr} + L_{rr}))L_{ss} - N\phi(L_{ss} + L_{sr} + L_{rr})L_{ss}$$
$$+ \beta \sum_{i=1}^{N}((M^i_{ss})^2 + \frac{1}{2}M^i_{ss}M^i_{sr} + \frac{1}{4}(M^i_{sr})^2)$$
$$\frac{dL_{sr}}{dt} = -(\mu_L + D_L(L_{ss} + L_{sr} + L_{rr}))L_{sr} - N\phi(L_{ss} + L_{sr} + L_{rr})L_{sr}$$
$$+ \beta \sum_{i=1}^{N}(M^i_{ss}M^i_{rr} + \frac{1}{2}(M^i_{ss}M^i_{sr} + M^i_{sr}M^i_{rr} + (M^i_{sr})^2))$$
$$\frac{dL_{rr}}{dt} = -(\mu_L + D_L(L_{ss} + L_{sr} + L_{rr}))L_{rr} - N\phi(L_{ss} + L_{sr} + L_{rr})L_{ss}$$
$$+ \beta \sum_{i=1}^{N}((M^i_{rr})^2 + \frac{1}{2}M^i_{rr}M^i_{sr} + \frac{1}{4}(M^i_{sr})^2)$$

Back to the future

- To try and find data which may lead to a more appropriate moment closure
- To incorporate treatment into the model
- To study the effects of optimised control on a treated population

You thought I'd forgotten...

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