Modelling Centrifugal Compressors

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Introduction

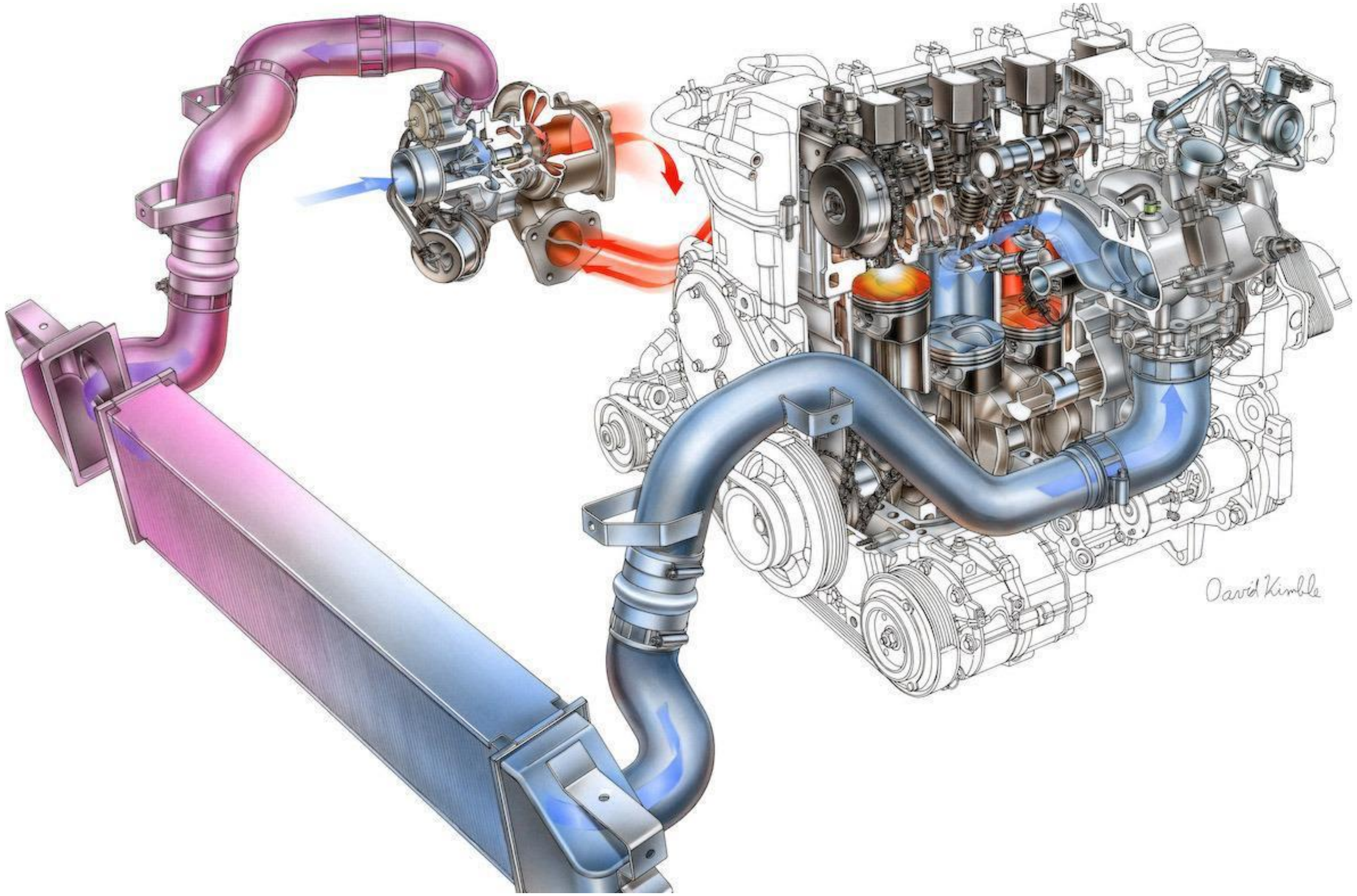
Aim:

- To find a simple analytical model for centrifugal compressors in turbochargers

Why?

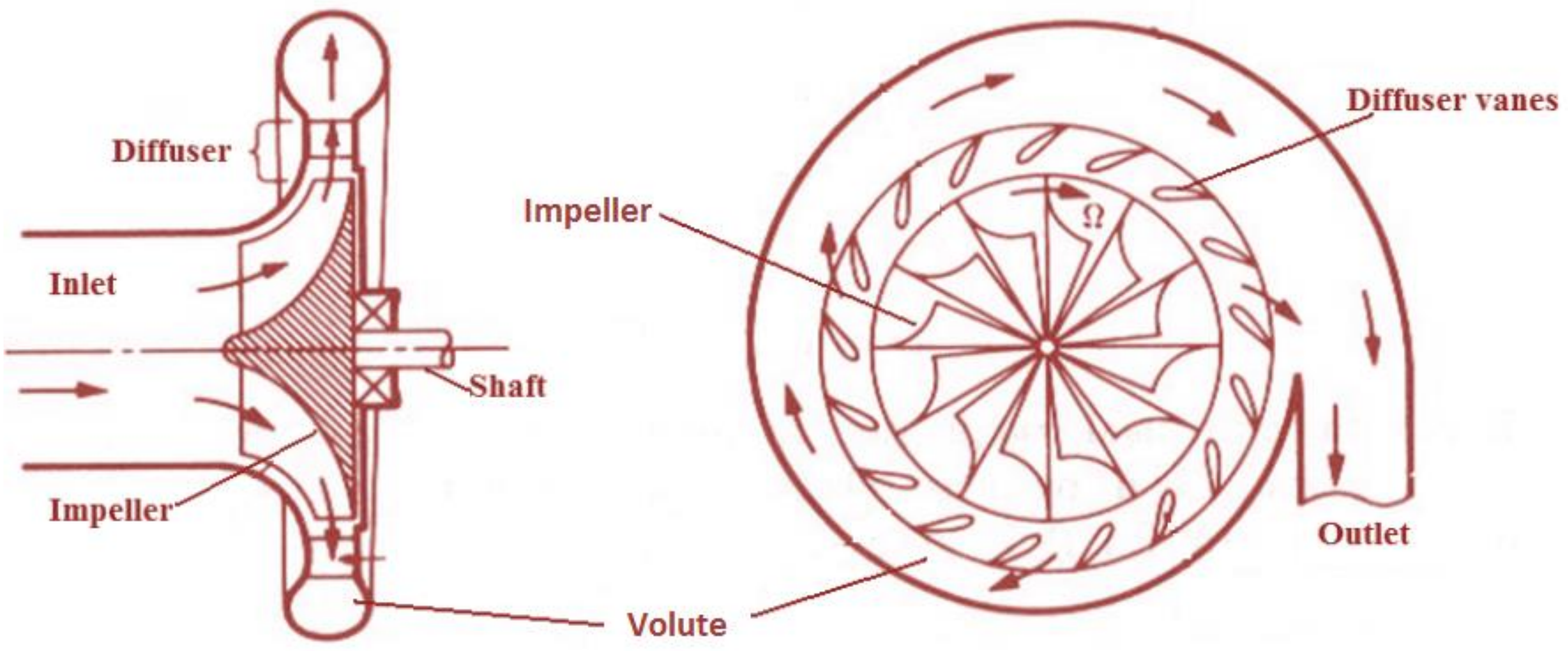
- In 2013, 35 million vehicles licensed for use in Great Britain alone
- In 2012, total CO_2 emissions from cars in UK was 63 million tonnes
- Improvements to turbochargers leads to more fuel efficient cars

Turbocharged Car Engine



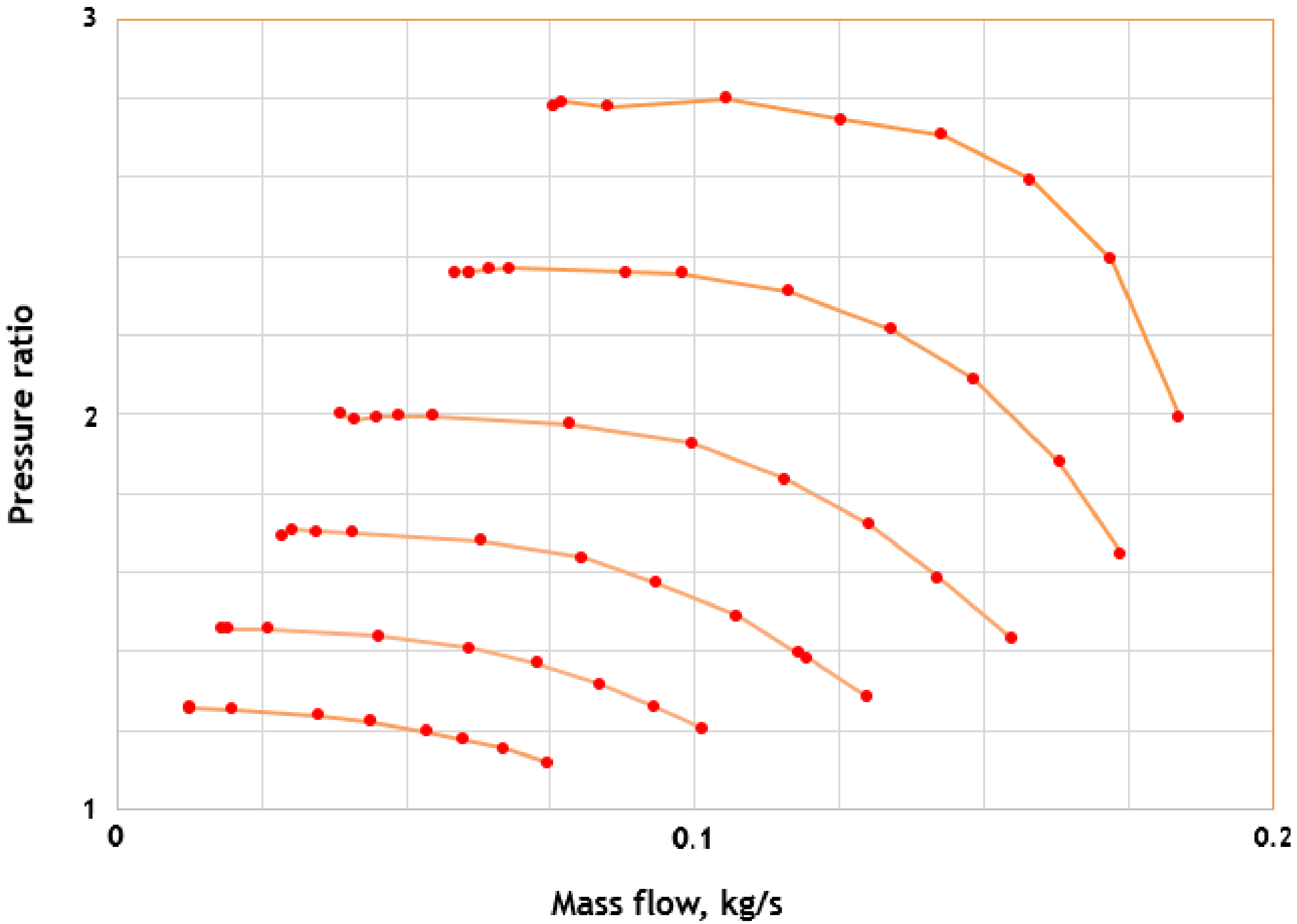
http://www.masmaservices.com/Engine-Turbocharger-Diagram.html

Centrifugal Compressor

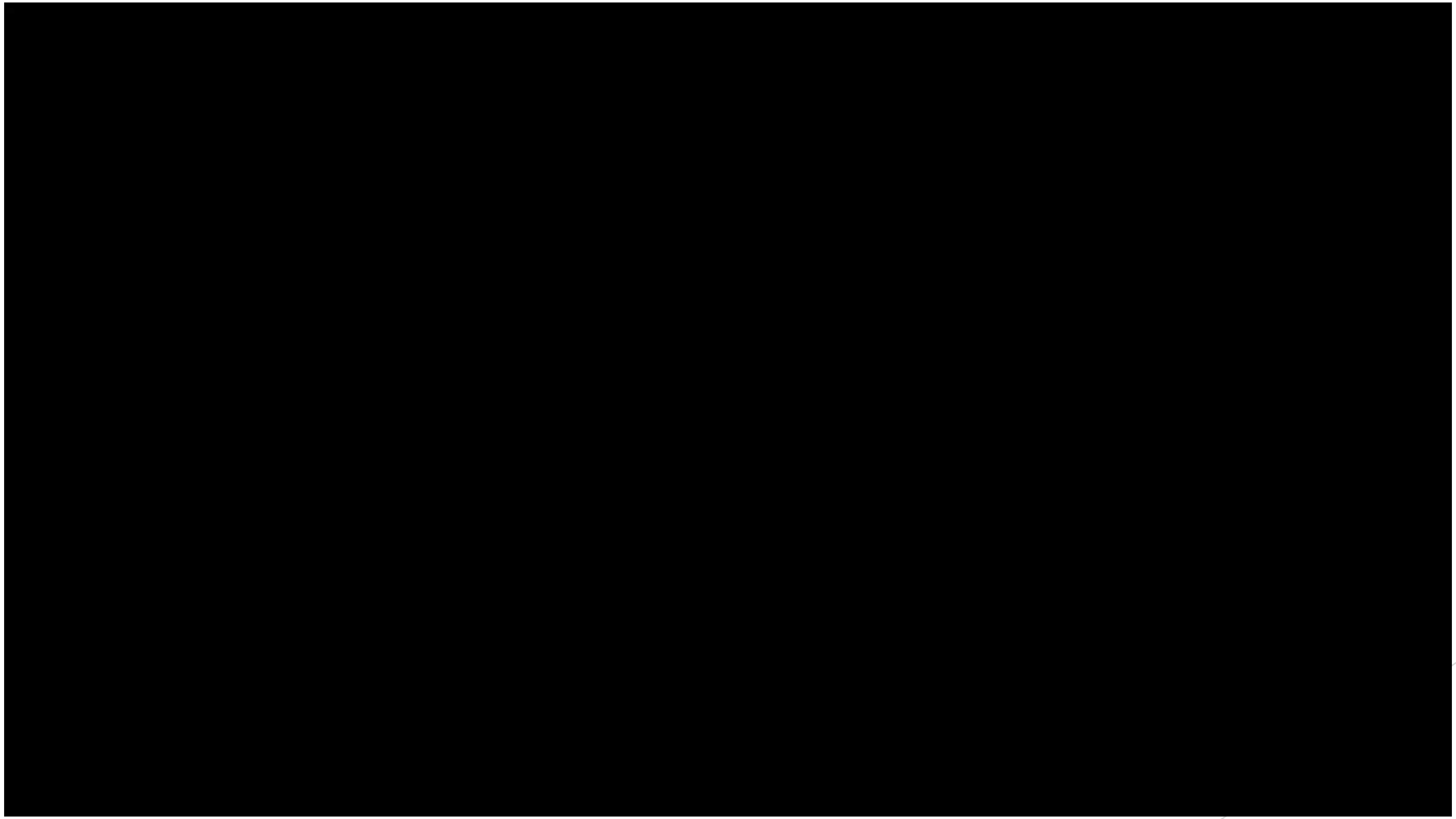


http://nptel.ac.in/courses/101101058/downloads/Lec-31ppts.pdf

Compressor Map



Surge



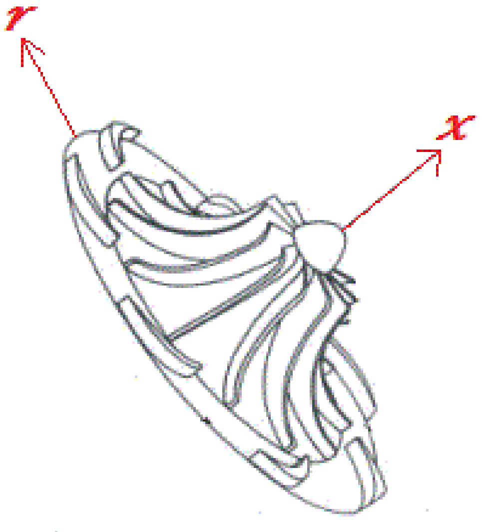
Equations of Motion

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho(\mathbf{u} \otimes \mathbf{u})) + 2\rho(\boldsymbol{\Omega} \times \mathbf{u}) + \rho(\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})) = -\nabla p \tag{2}$$

$$p = \kappa \rho^{\gamma} \tag{3}$$

Steady-state Equations



$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{\partial}{\partial x}(\rho u_x) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r^2) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_r u_\theta) + \frac{\partial}{\partial x}(\rho u_r u_x) - \frac{\rho u_\theta^2}{r} - 2\rho\Omega u_\theta - \rho\Omega^2 r = -\frac{\partial p}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r u_\theta) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta^2) + \frac{\partial}{\partial x}(\rho u_\theta u_x) + \frac{\rho u_r u_\theta}{r} + 2\rho\Omega u_r = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho u_x u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta u_x) + \frac{\partial}{\partial x}(\rho u_x^2) = -\frac{\partial p}{\partial x}$$

$$p = \kappa \rho^\gamma$$

http://nptel.ac.in/courses/Webcourse-contents/IIT-KANPUR/machine/ui/Course_home-lec6.htm

Impeller Geometry

http://www.blue-sky-technologies.com/impeller-seal-repair/

https://dir.indiamart.com/chennai/fan-impeller.html

Diffuser Geometry

http://www.technologie-entwicklung.de/Gasturbines/Monocopter/Turbine_Details/turbine_details.html

Model Simplification

- Assumptions:
  - Impeller: $u_\theta = 0$
  - Diffuser: $\frac{\partial}{\partial \theta} = 0$

- Average over x : $\bar{G} = \frac{1}{h(r)} \int_0^{h(r)} G \; dx$

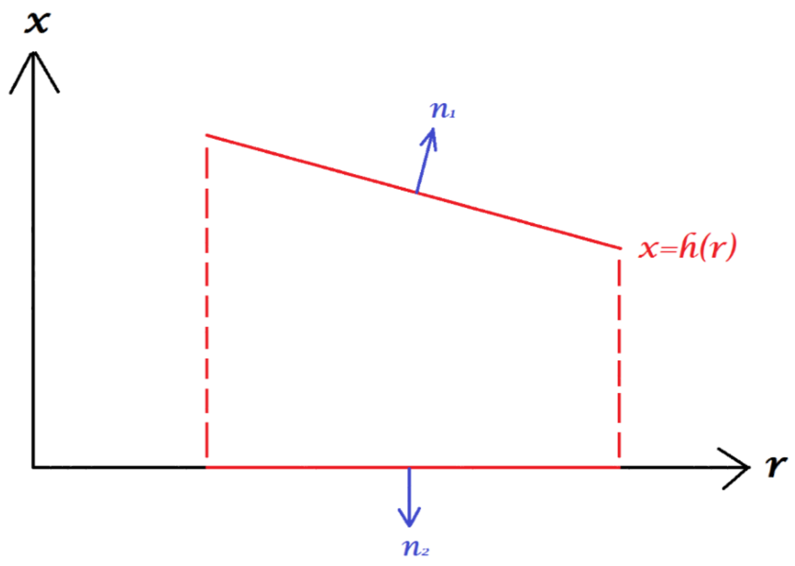
Averaging

$$\int_0^h \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{\partial}{\partial x}(\rho u_x)\ \ dx = 0$$

$$\Longrightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left(\int_0^h r\rho u_r\ \ dx\right) - \rho u_r\Big|_h \frac{\partial h}{\partial r} + \rho u_x\Big|_0^h = 0$$

$$\Longrightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r}) + \rho\left(-u_r\frac{\partial h}{\partial r} + u_x\right)\Big|_h - \rho u_x\Big|_0 = 0$$

$$\Longrightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r}) = 0$$



Averaged equations

Impeller:

$$\frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r}) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r^2}) - h\bar{\rho}\Omega^2 r = -\frac{\partial}{\partial r}(h\bar{p}) + p\Big|_h \frac{\partial h}{\partial r}$$

$$\bar{p} = \kappa\overline{\rho^\gamma}$$

Diffuser:

$$\frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r}) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r^2}) - \frac{h\overline{\rho u_\theta^2}}{r} = -h\frac{\partial \bar{p}}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rh\overline{\rho u_r u_\theta}) + \frac{h\overline{\rho u_r u_\theta}}{r} = 0$$

$$\bar{p} = \kappa\overline{\rho^\gamma}$$

Resulting Equations

Impeller:

$$2\pi r h \rho u_r = \dot{m} = \text{const.}$$

$$\frac{u_r^2}{2} + \frac{\kappa\gamma}{\gamma - 1}\rho^{\gamma-1} = \frac{\Omega^2 r^2}{2} + c_1$$

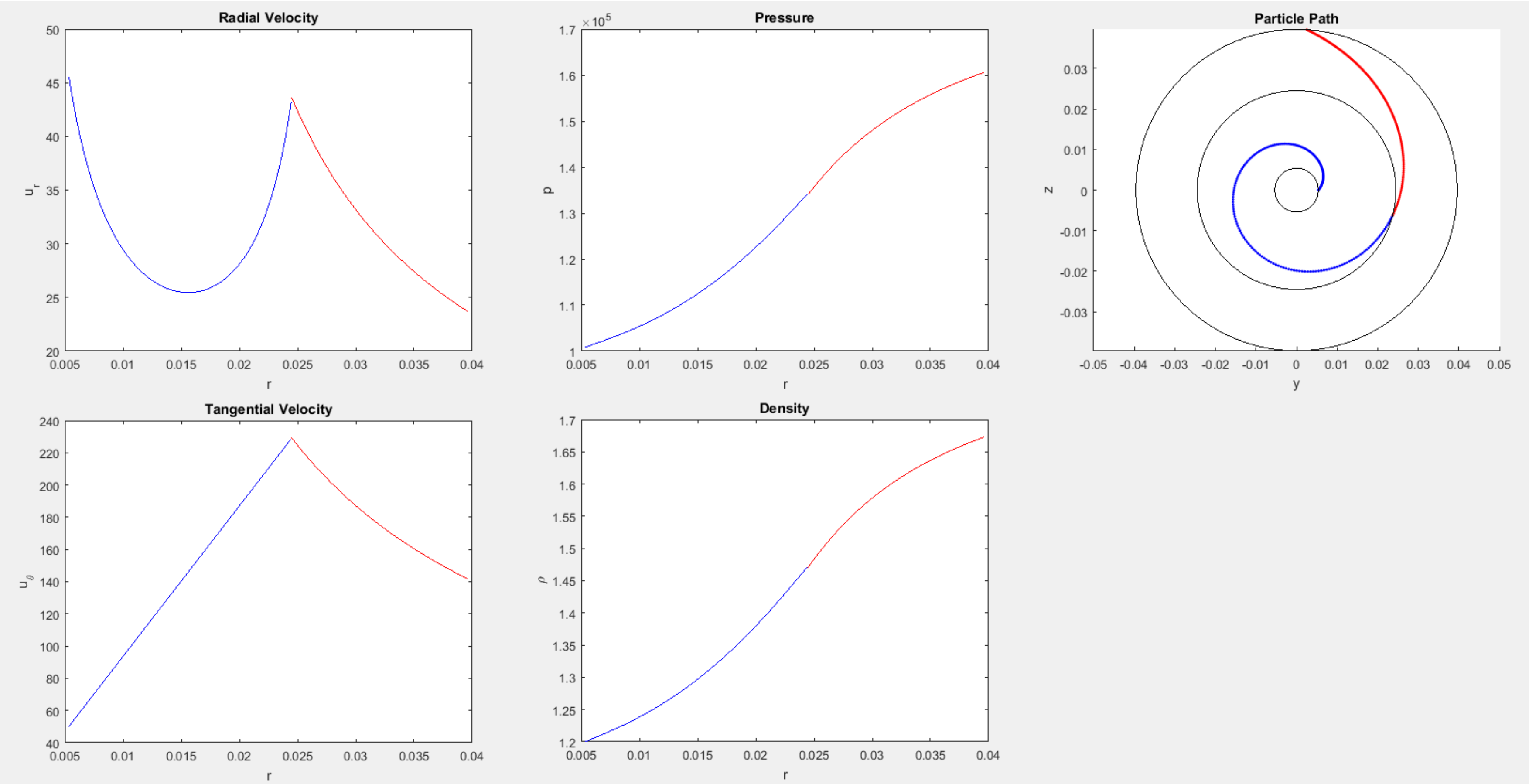
Diffuser:

$$2\pi r h \rho u_r = \dot{m} = \text{const.}$$

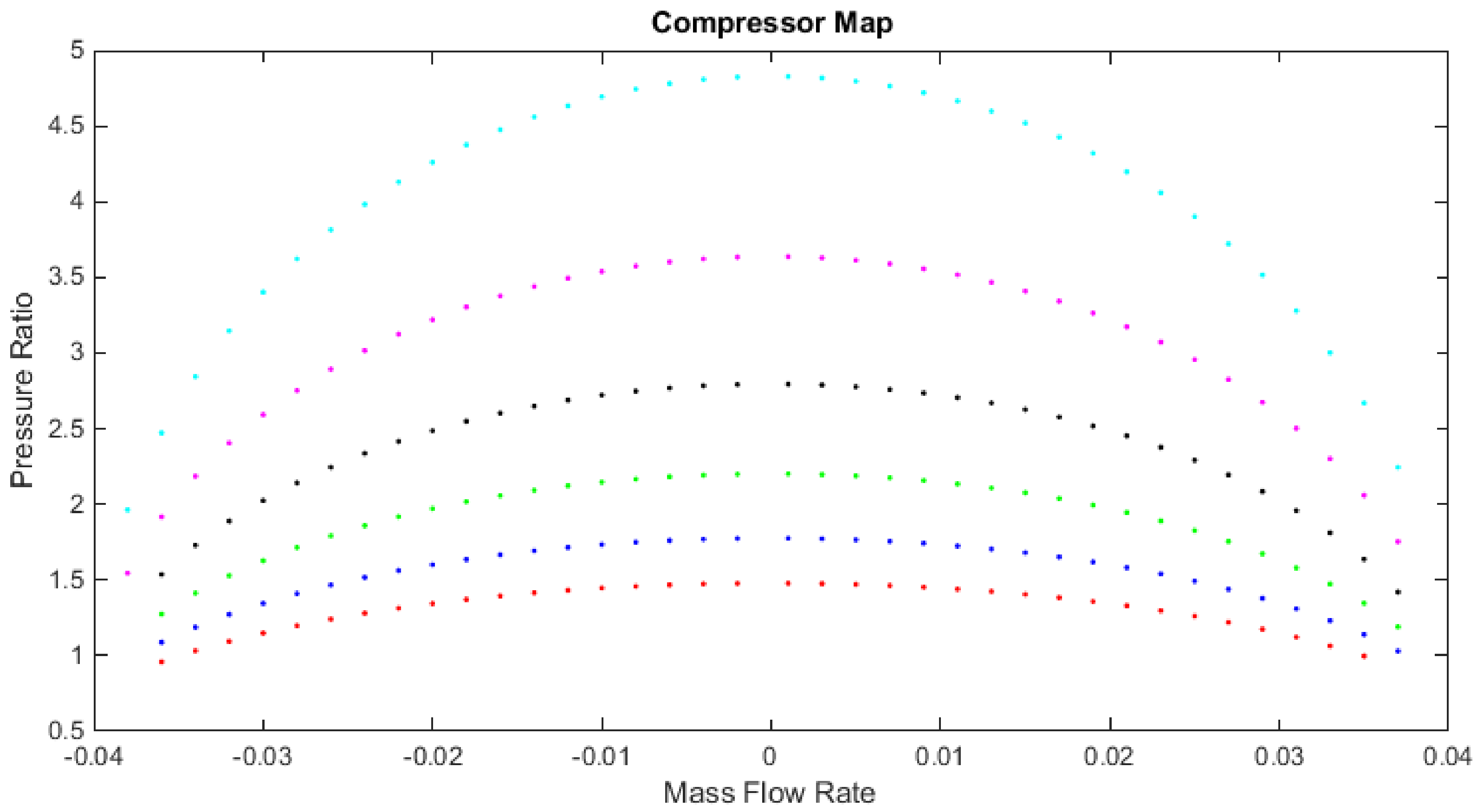
$$u_\theta = \frac{c}{r}$$

$$\frac{u_r^2}{2} + \frac{\kappa\gamma}{\gamma - 1}\rho^{\gamma-1} = -\frac{u_\theta^2}{2} + c_2$$

Results for given Inlet Conditions



Computed Compressor Map



Conclusions

- Surge is an important unstable phenomenon to model
- We can create a simple 1D model that captures most of the compressor flow dynamics

Future Work:

- Removing isentropic assumption
- Adding back-pressure
- Bifurcation analysis