Adaptive Sampling for Imaging

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February 1, 2019

Agenda

Problem Recap and Data

Scanning a battery to determine presence and distribution of materials.

From full scans, we observe $A_{full} \in \mathbb{R}^{n_1 \times n_2}$ (matrix of absorptions of n_1 energies at n_2 scanned pixels).

Aim: to find a reduced scanning pattern which allows us to recover A_{full}.

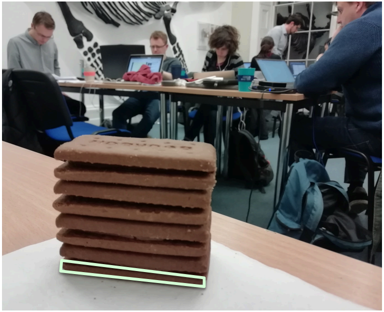
Problem Recap and Data

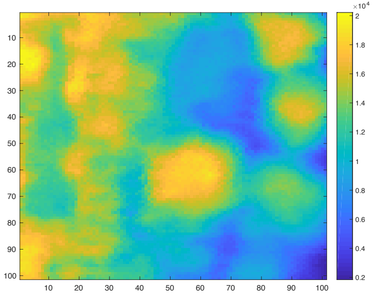
If the sample contains k components, we can approximate A_{full} by:

$$A_{full} \approx U_{spectral} C_{spectral},$$

where $U_{spectral} \in \mathbb{R}^{n_1 \times k}$ are the spectra of the materials and $C_{spectral} \in \mathbb{R}^{k \times n_2}$ are the coefficients for each pixel

Problem Recap and Data





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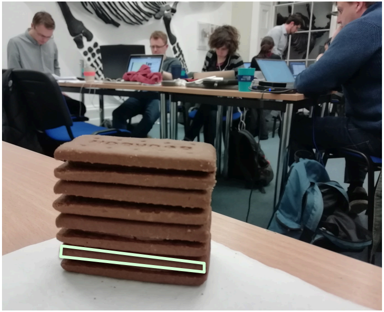

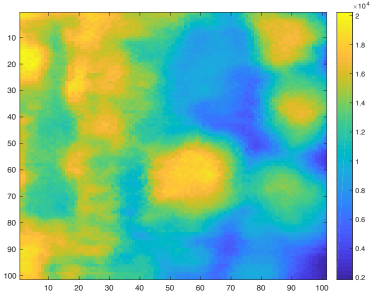

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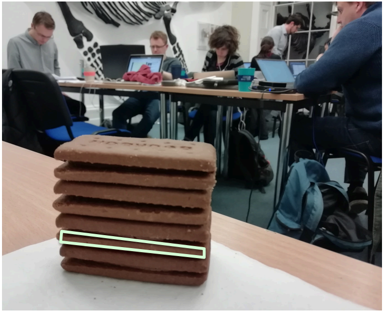

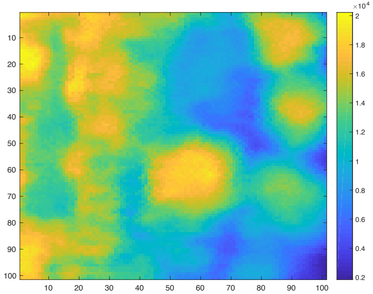

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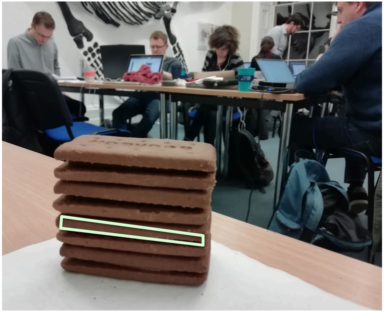

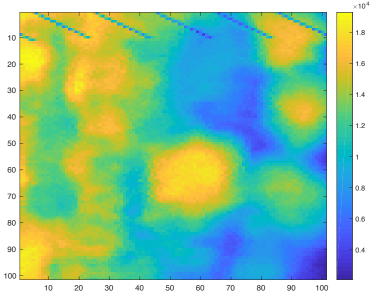

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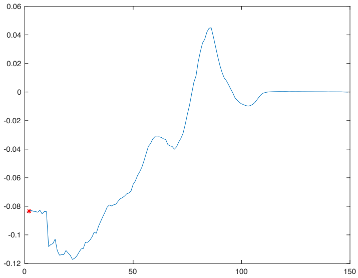

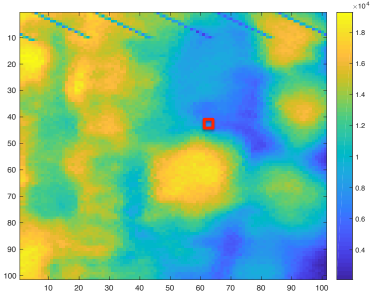

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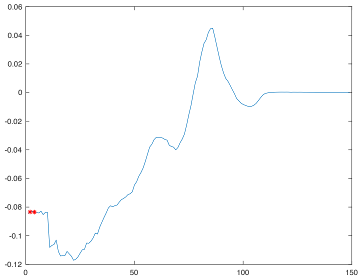

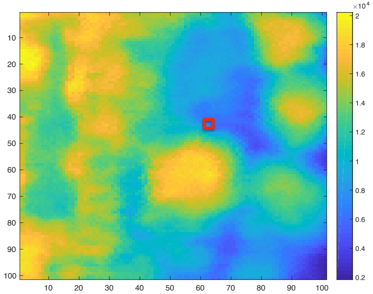

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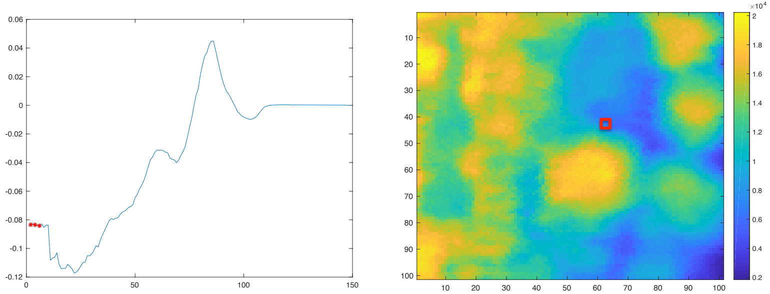

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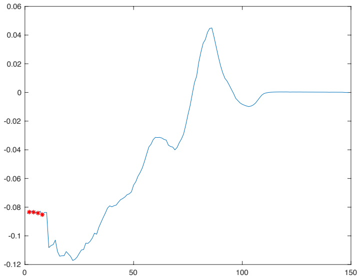

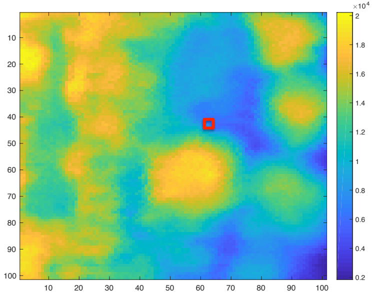

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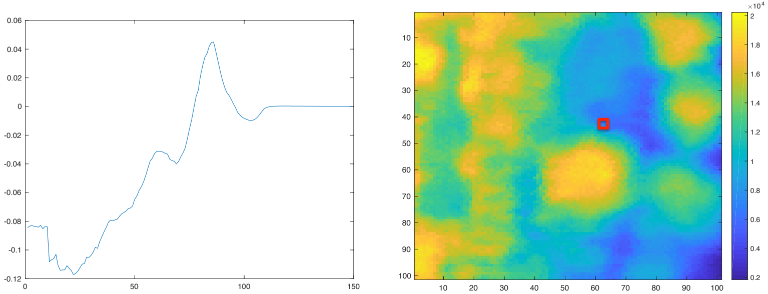

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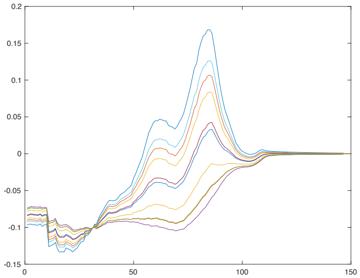

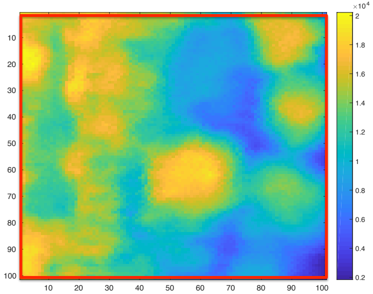

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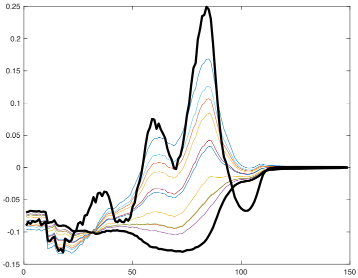
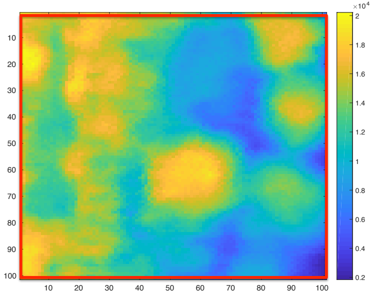
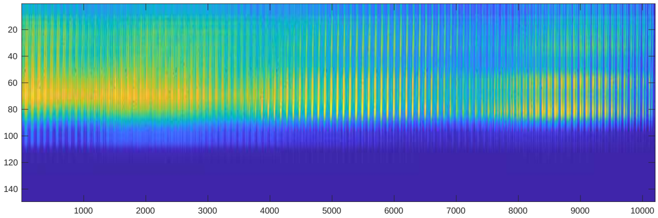




Multi-resolution Sampling Approach using DEIM

1. Use SVD on a coarse resolution scan A_{coarse} of all the energies (aggregated pixels) to identify a matrix U_0 which spans the same space than U_{spectral}.
2. Use DEIM to identify the important energies. High resolution scan (in all pixels) will just be performed for these energies. Compute $C_1 \approx C_{\text{SVD}}$ by imposing $A(p_k, :) = U_0(p_k, :) C_1 \; \forall k$.
3. Use dictionary of spectra to identify which materials are in the sample.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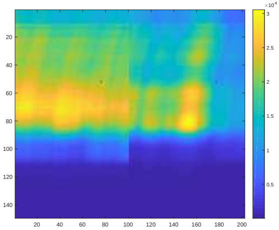
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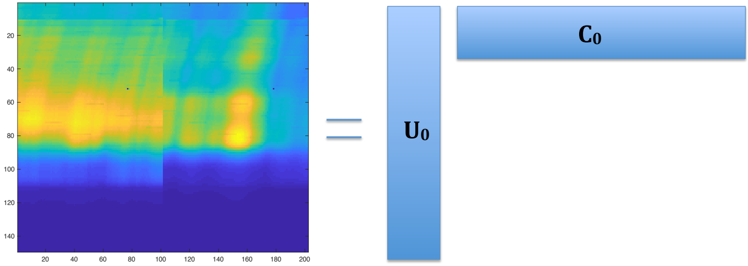

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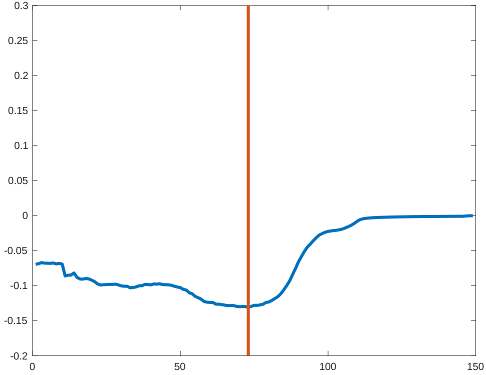

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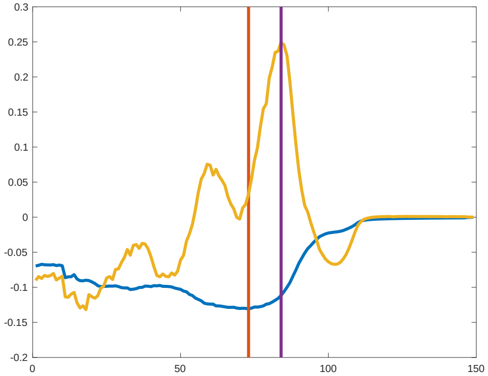

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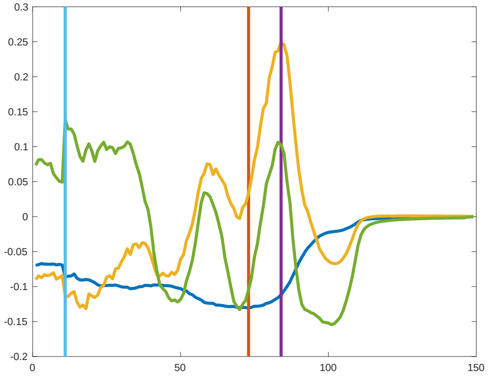

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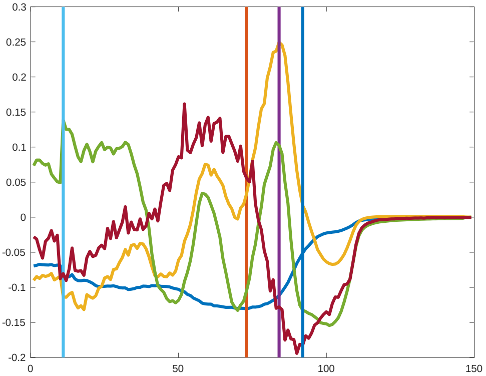

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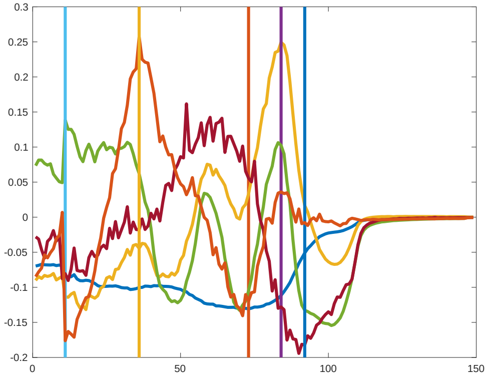

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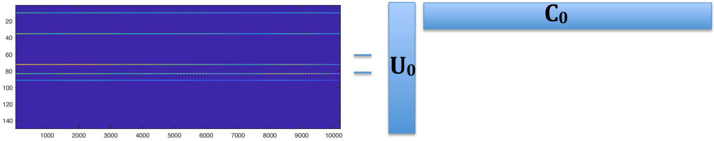
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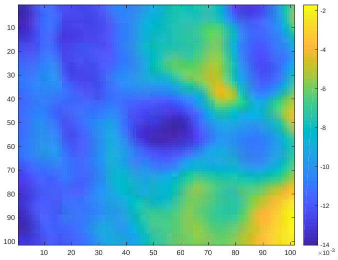
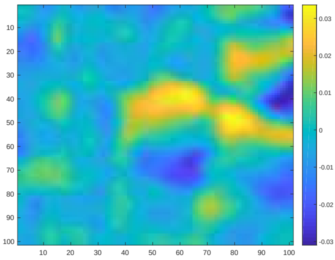
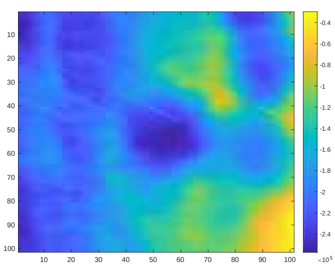


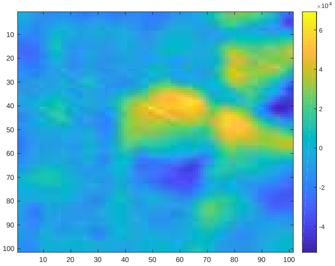
(a) $U_{\mathrm{SVD}}(:,1)$



(b) $U_{\mathrm{SVD}}(:,2)$



(c) $U_0(:,1)$



(d) $U_0(:,2)$

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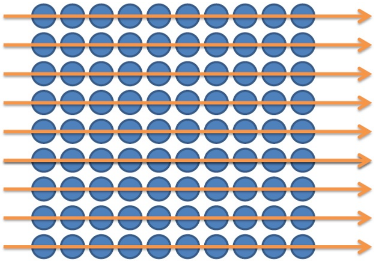
Matrix Completion Approach

Current method: raster scan through battery

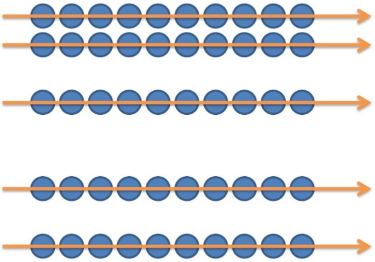


Matrix Completion Approach

Current method: raster scan through battery

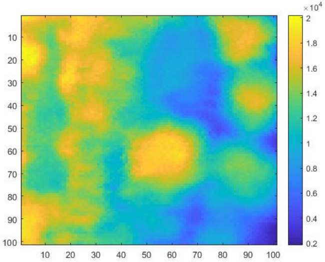
Could we only scan random lines instead?

Then we need a way to infer the gaps

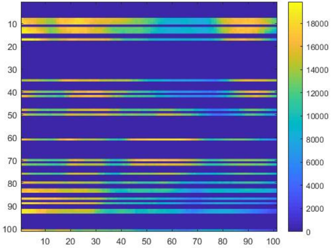


Matrix Completion Approach

A 2D scan of a sample, at fixed energy...

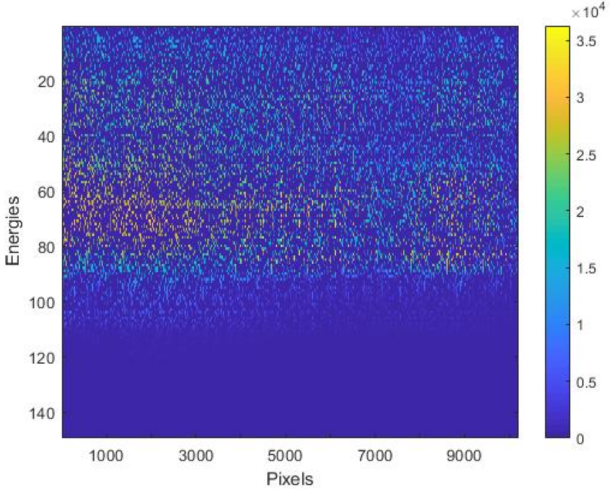


...and after randomly removing 80% of the rows:



Matrix Completion Approach

Removing 80% of the rows in each image, and combining the results into one large matrix:



Matrix Completion Approach

Matrix completion problem:

- $M \in \mathbb{R}^{n_1 \times n_2}$ of rank r;
- We know m elements $M_{ij},\ (i,j) \in \Omega$;
- $\Omega \subset \{1, ..., n_1\} \times \{1, ..., n_2\}$ contains the indices of known elements.

Can we find M_{ij} for $(i,j) \notin \Omega$?

For most matrices, this can be achieved by using an iterative algorithm[2] to approximately solve:

$$\begin{aligned} \text{minimize} \quad & ||X||_* \\ \text{s.t.} \quad & X_{ij} = M_{ij},\ (i,j) \in \Omega \end{aligned}$$

[2] Jian-Feng Cai, Emmanuel J. Candés, and Zuowei Shen. "A Singular Value Thresholding Algorithm for Matrix Completion". In: *SIAM Journal on Optimization* 20.4 (2010), pp. 1956–1982.

Matrix Completion Approach

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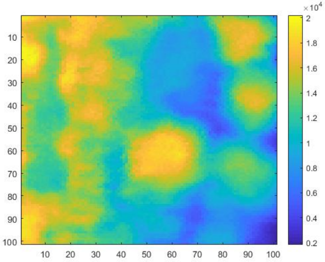
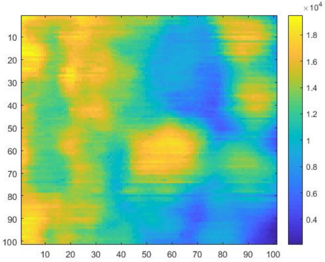


Reconstructed sample:



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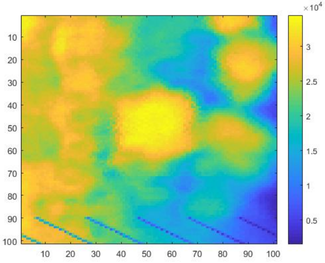

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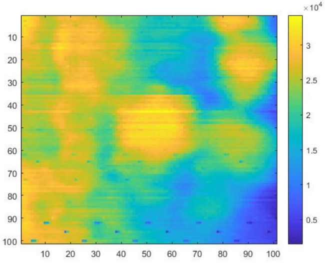

Ideas for Future Work

- Combining the two methods: undersampling in energy and space
- Extending to rotation of samples for 3D imaging

References

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