RISK MEASURES FOR CATASTROPHE INSURANCE

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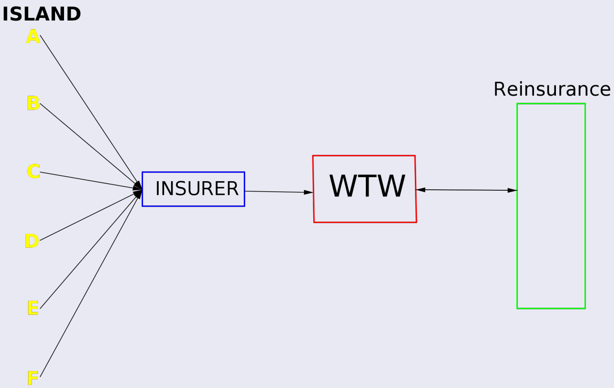
Overview

- Introduction
- Risk Measure properties
- Risk Allocation
- Conclusion
- Future work

Problem Statement

Sketch



Current method

Issue with Kreps' approach

Premiums currently calculated using Kreps' paper (1990) which may no longer be appropriate. As the approach focuses more on the individual premium rather than the combined premium and as a result, the islands might have to pay more than their actual risk. Therefore, we are looking for a fair way in which the premiums can be allocated amongst the islands.

Risk Measure

Suppose $X_1, X_2, \cdots, X_n$ are the losses of each islands and ρ be the measure of the risk of the islands.

Properties

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- Monotone: If $X_1 \leq X_2$, then

$$\rho(X_1) \geq \rho(X_2)$$

Entropic Risk Measure

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Mathematical Expression

The entropic risk measure with the risk aversion parameter $\theta > 0$ is defined as

$$\rho^{\text{ent}}(X) = \frac{1}{\theta} \log \left(\mathbb{E} \left[e^{-\theta X} \right] \right)$$

Joint price of the premium

Insurance Premium

Using a Taylor expansion:

$$\rho^{\text{ent}}(X) = \frac{1}{\theta} \log\left(\mathbb{E}\left[e^{-\theta X}\right]\right)$$
$$\approx \mathbb{E}(X) + \frac{\theta}{2}\,\text{Var}(X)$$

Allocation Rules

Let $\rho(X_i)$ be the risk measure for island i, $i \in N = \{1, 2, ...n\}$, and $\rho(X_S)$ be the risk measure for a subset S of N. In particular $\rho(X_N)$ represents the measure of combined risk across all islands.
We consider various allocation rules mapping these risk measures into premiums for individual islands as presented in Jorion, P. (2007) and Balog, D. et al (2017)

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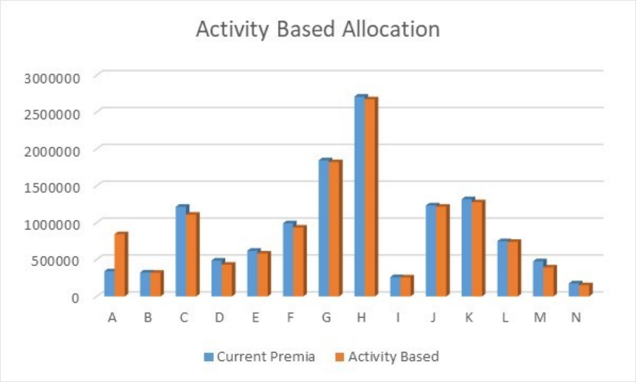
Activity Based Method

$$\phi_i^{AB}(X_N^{\rho}) = \frac{\rho(X_i)}{\sum_{j \in N} \rho(X_j)} \rho(X_N)$$



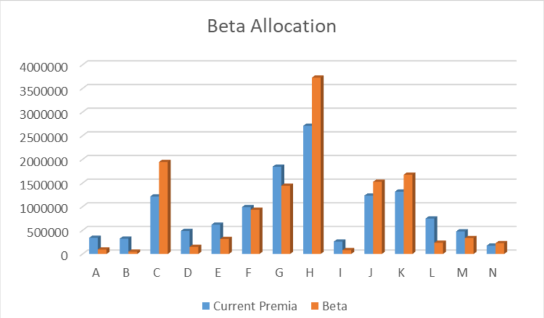
Beta Method

$$\phi_i^{\beta}(X_N^{\rho}) = \frac{\beta_i}{\sum_{j \in N} \beta_j} \rho(X_N)$$

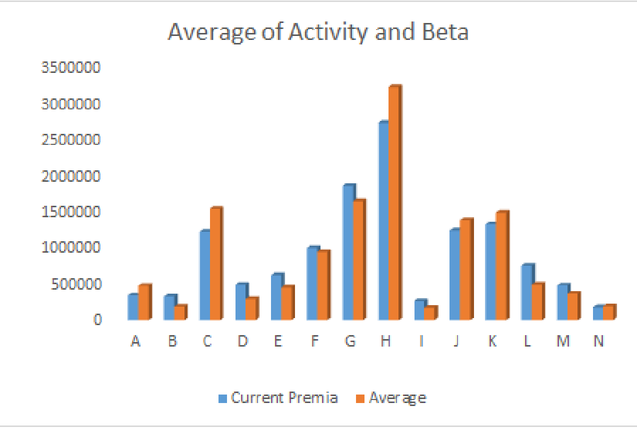
where

$$\beta_i = \frac{Cov(X_i, X_N)}{Cov(X_N, X_N)}$$

$Cov(X_i, X_N)$: covariance between the random variables describing the financial position of subunit $i \in N$ and the main financial unit.

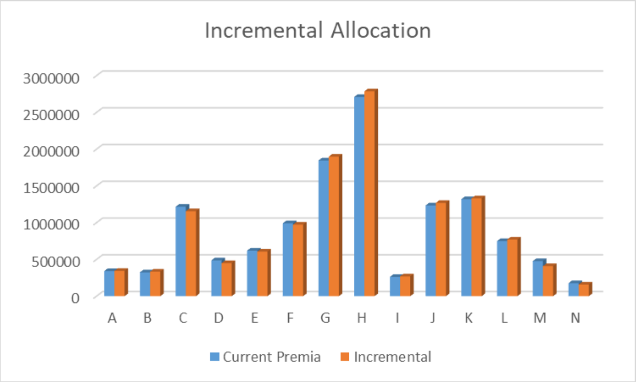


Average of Activity and Beta



Incremental Method

$$\phi_i^I(X_N^\rho) = \frac{\rho(X_N) - \rho(X_{N\setminus\{i\}})}{\sum_{j\in N}(\rho(X_N) - \rho(X_{N\setminus\{j\}}))}\rho(X_N)$$



Conclusion

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The beta allocation approach tends to be better as small islands will be allocated with small premium as their risks have lower correlation with the group's. Finally, the insurer can decide to use the activity based allocation method when the individual risk is of interest. If correlations are significant, for example in estimating the chance of a reinsurance claim, they might also incorporate the beta allocation rule.

Future Directions

Questions to consider

CCRIF require reinsurance to offload some of their risk. This raises new questions:

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- How much should they reinsure?
- What amount should they expect to pay?
- How rubust is the average allocation method? Is this method consistent if other financial data is considered?
- How reluctant will the reinsurer be to insure risks correlated to US?

Risk measures may offer ways to approach these questions.

References

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