Physical and Numerical Analysis of Hurricanes

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Challenge

Goal: Assign a probability that a target region, for which we have no historical data, would be hit by a Tropical cyclone.

Data:

- Track in Latitude/Longitude
- Max wind speed
- Central Pressure

Issue:

- Probability of exact hypothetical track is extremely small
- Hurricane dynamics are complex and not fully understood

Overview of method - Partitioning approach

How to estimate the Hurricane trajectory?

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = c + U \tag{1}$$

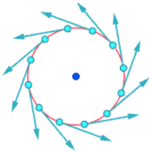
- c: Advection due to Coriolis Force. Solve vorticity equation in absence of background flow.
- U: Advection due to large-scale atmospheric wind flow. Fit each hurricane track with an ellipse function and obtain an empirical probability distribution $F(u)$.

Inverse Problem: $\mathbf{v}, c \rightarrow U \rightarrow F^{-1}(U)$

Vorticity equation

We consider a 2D Navier-Stokes equation in a barometric framework.

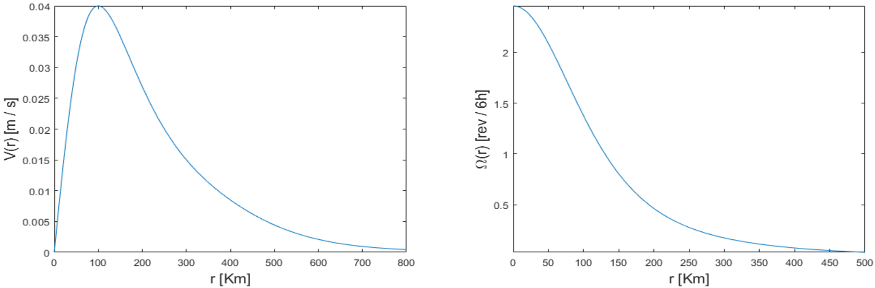
$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \beta v = 0 \tag{2}$$



- $\mathbf{u} = (u, v)$ velocity vector of hurricane
- $\beta = \frac{df}{dy}$ Derivative of Coriolis parameter
- $\zeta = \mathbf{k} \cdot (\nabla \times \mathbf{u})$ relative vorticity

Initial State: symmetric vortex at the origin with an imposed tangential and angular velocity vector $V(r), \Omega(r) = V(r)/r$.

Tangential and Angular velocity



Over the time, the hurricane develops asymmetries due to the interaction with the ambient flow that generates itself.

Partitioning problem

We write $\zeta = \zeta_s + \Gamma$ to split ζ into an axisymmetric part for the core of the hurricane which just rotates, and an asymmetric correction term Γ. After some simplifications we can then split the vorticity eqn into two pieces:

$$\frac{\partial \zeta_s}{\partial t} + \mathbf{c}(t) \cdot \nabla \zeta_s = 0 \quad (3)$$

$$\frac{\partial \Gamma}{\partial t} = -\mathbf{u} \cdot \nabla(\, \Gamma + f \,) \quad (4)$$

Streamfunction Ψ

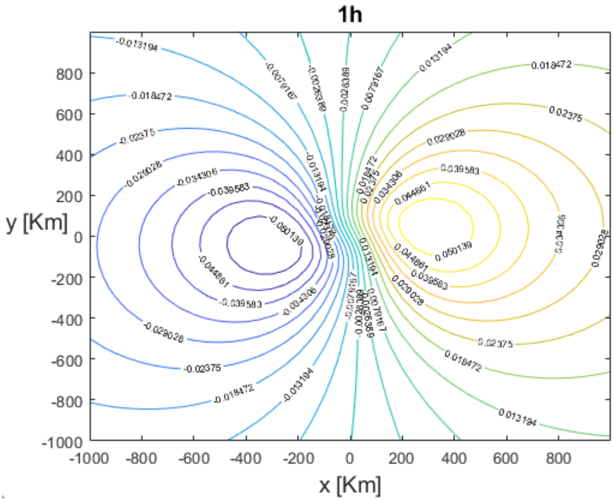
In cylindrical coordinates (r, θ), the equation for Γ is written as:

$$\left[\frac{\partial}{\partial t} + \Omega(r) \frac{\partial}{\partial r} \right] (\Gamma + \beta y) = 0 \qquad (5)$$

From Γ, we can solve the Poisson equation $\nabla^2 \Psi = \Gamma$ to obtain the Streamfunction. It follows that:

$$\Psi = \Psi_1(r,t) cos(\theta) + \Psi_2(r,t) sin(\theta) \qquad (6)$$

The Streamfunction contour levels represent the direction along which the vortex of the hurricane moves.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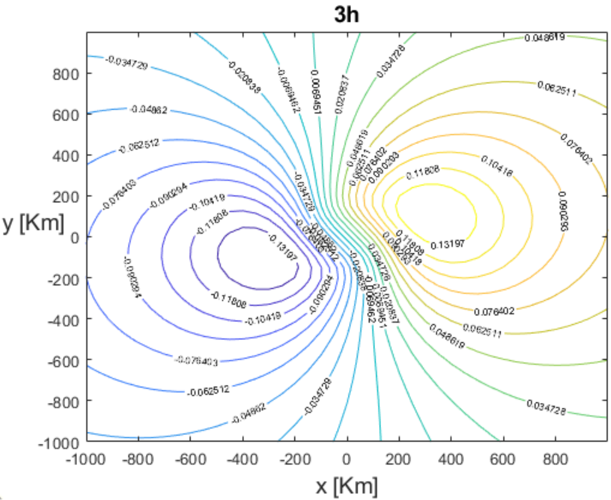
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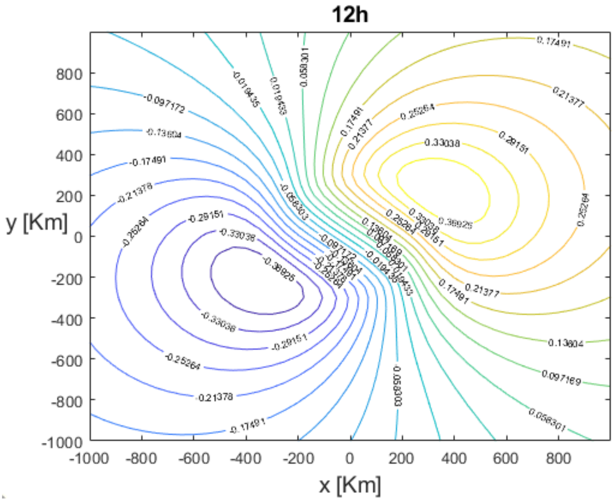
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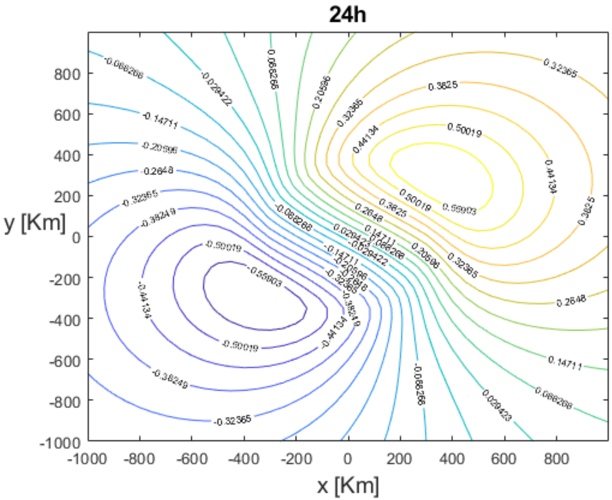
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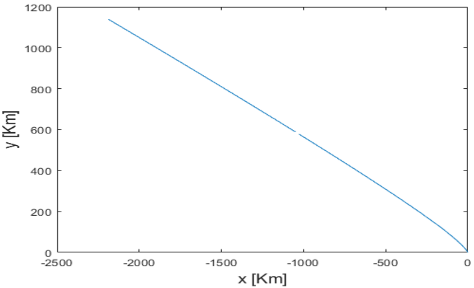
Solution for vortex speed

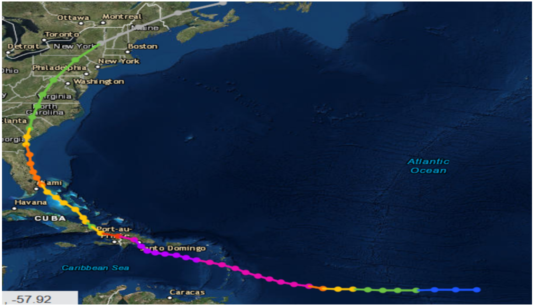
Finally the vortex speed is given by:

$$\left[X(t), Y(t)\right]^T = \begin{bmatrix} \frac{1}{2}\beta \int_0^\infty r\left[t - \frac{sin\{\Omega(r)t\}}{\Omega(r)}\right] dr \\ \frac{1}{2}\beta \int_0^\infty r\left[1 - \frac{1-cos\{\Omega(r)t\}}{\Omega(r)}\right] dr \end{bmatrix} \quad (7)$$



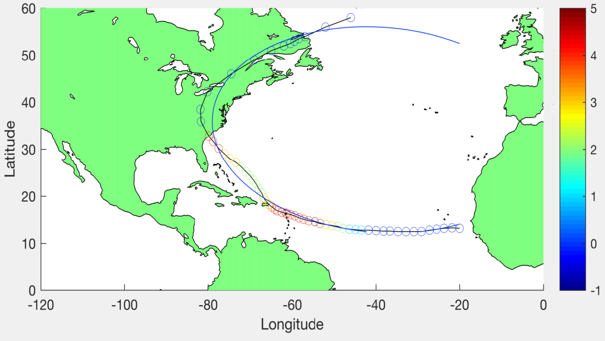
In addition to the component c, the other cause of the large scale hurricane motion is the environment velocity U.

We compute different U by fitting the Hurricane tracks provided by the Historical data to an ellipse. We also obtain an empirical distribution $F(U)$ to sample from.



Inverse Problem

If we have an idea of a potential hurricane speed that might hit a region, we can extract the environment field U from eq. 1 and obtain the corresponding probability from its CDF.



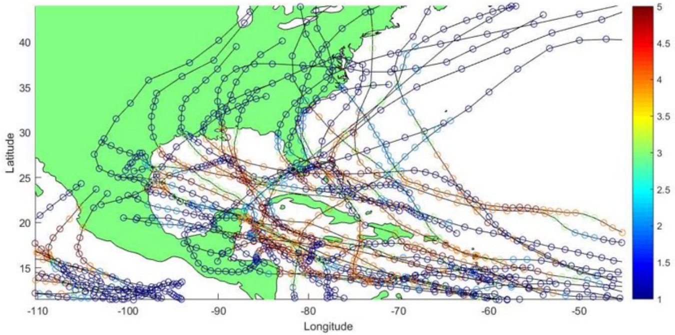
Summary

- Geophysics suggests that hurricanes move due to two distinct mechanisms.
- The interaction between the hurricane vortex and the Coriolis force term can be estimated analytically, while large-scale atmospheric flows additionally steer it around the Atlantic basin.

For future work:

- Combine the two elements to produce a semi-analytic model that estimates whole tracks
- We should find more regularity in the distribution of these parameters that leads to more reliable estimates for as-yet unseen tracks

Thank you for your attention!



References

Roger K.Smith and Wolfgang Ulrich (1990)
An Analytical Theory of Tropical Cyclone Motion Using a Barotropic Model
Meteorological Society.

Roger K.Smith and Wolfgang Ulrich and Gary Dietachmayer (1990)
A Numerical Study of Tropical Cyclone motion using a barotropic model, I: The role of vortex asymmetries
Meteorological Society.

Roger K.Smith and Wolfgang Ulrich (1991)
A Numerical Study of Tropical Cyclone motion using a barotropic model, II: Motion in spatially-varying large-scale flows
Meteorological Society.

Solution for Environment Vorticity Equation

The equation 5 is integrated to give the solution:

$$\Gamma(r,\theta,t) = \zeta_1(r,t)cos(\theta) + \zeta_2(r,t)sin(\theta) \tag{8}$$

where $\zeta_1 = -\beta r sin\Omega(r)t$ and $\zeta_2 = -\beta r[1 - cos\Omega(r)t]$.