A very brief overview of population dynamics models

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Model types

Population dynamics models can be:

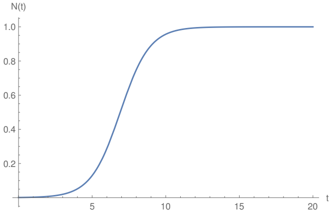
- Continuous / Discrete
- Deterministic / Stochastic
- Well-mixed / Spatial / Network
- Single-type / Muli-type

Logistic model

Continuous, deterministic, well-mixed, single-type.

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

N - size of population, r - growth rate, K - carrying capactiy



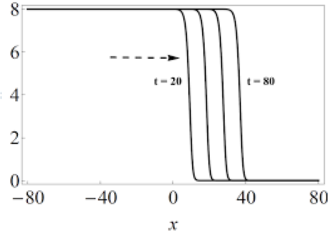
Unstable steady state at $N = 0$, stable steady state at $N = K$.

Fisher KolmogorovPetrovskyPiskunov (FKPP) model

Continuous, deterministic, spatial, single-type.

$$\frac{\partial N}{\partial t} = rN\left(1 - \frac{N}{K}\right) + D\frac{\partial N}{\partial x^2}$$

D - diffusion rate



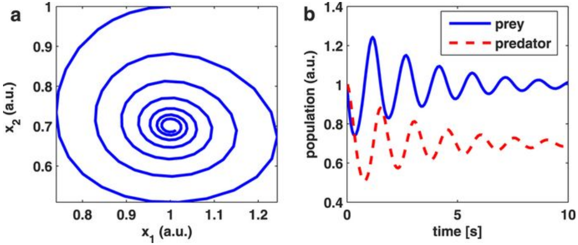
Travelling wave solutions propagating into unstable $N = 0$ state.

Generalised Lotka-Volterra model

Continuous, deterministic, well-mixed, multi-type.

$$\frac{dN_i}{dt} = N_i f_i(\boldsymbol{N}), \quad \boldsymbol{f} = \boldsymbol{r} + A\boldsymbol{N}$$

N_i - size of population i, $\boldsymbol{r}$ - growth rates, A - community matrix



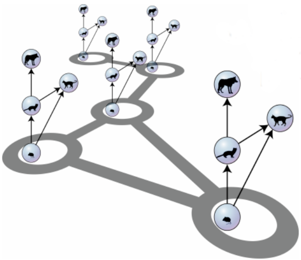
Point attractors, limit cycles, chaos, etc.

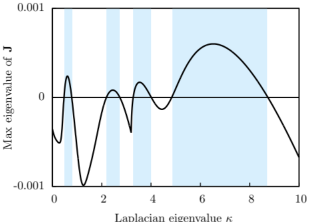
Metapopulation models

Continuous, deterministic, networked, multi-type.

$$\frac{d\boldsymbol{N}}{dt} = \boldsymbol{N} \circ (\boldsymbol{r} + A\boldsymbol{N}) + D\Delta\boldsymbol{N} \,,$$

Δ - graph Laplacian (transport matrix), $\circ$ - Hadamard product





Stability depends on population-network interaction.

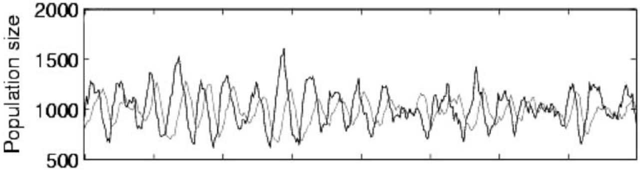
Stochastic Lotka Volterra models

Discrete, stochastic, well-mixed, multi-type.

$$X_i \xrightarrow{r_i} 2X_i \quad X_i \xrightarrow{d_i(\boldsymbol{X})} \varnothing$$

$$\Rightarrow \quad dN_i = N_i(r_i - d_i)\,dt + \sqrt{\frac{N_i(r_i + d_i)}{V}} dW_i \,.$$

X_i - type i, r_i - birth rate, $d_i(\boldsymbol{x})$ - death rate, V - volume



Quasicycles, extinction, etc.