Small Catchments as Tree-like Structures

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The Problem with Small Catchments

- Small catchments are areas with $<25km^2$
- They often have little or no reliable historic data to provide reliable prediction
- Flood estimation is crucial as there is less time to respond
- Local properties of small catchments mean it is difficult to gauge whether behaviour at one catchment can be attributed to another catchment...
- ...and consequently statistical approaches so far have been wildly inaccurate

Creating a Tree for a Small Catchment

- Use information about the environment in the catchment to create a tree (e.g. topography, soil content, urbanization, etc.)
- We hope this tree will be a "fingerprint" for the catchment

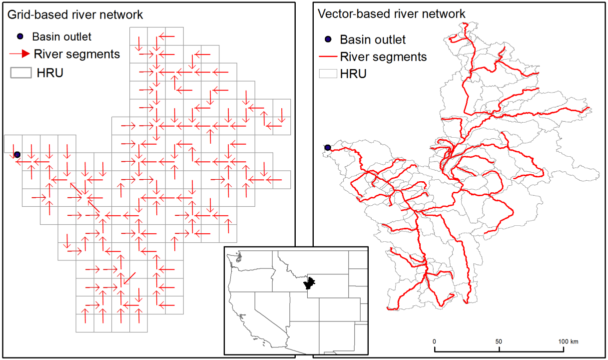


Figure: N. Mizukami et al, Geosci. Model Dev., 9, 2223–2238, 2016

Creating a Tree for a Small Catchment

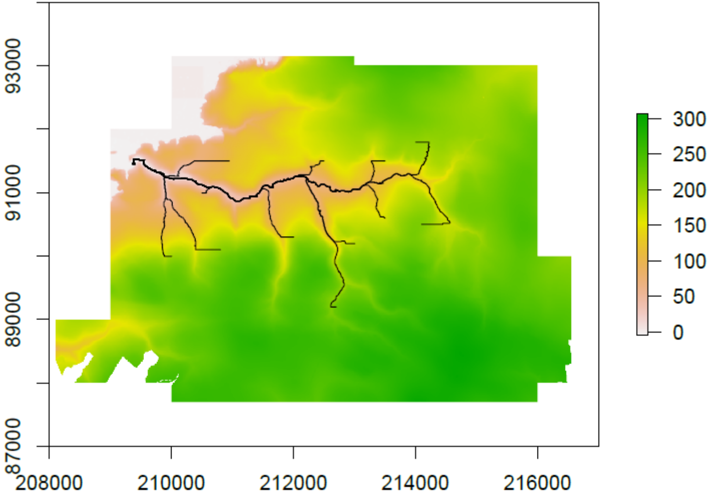
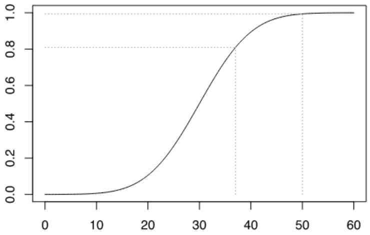
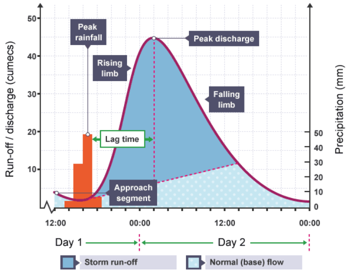


Figure: Topography of Boscastle

Catchment Drainage Profile and Hydrographs





- From the tree we can derive the *catchment drainage profile*, which is the proportion of rain particles whose initial distance from the root is less than t.
- The hydrograph can be obtained by differentiating the drainage profile with respect to time.

What will *bad* Rainfall look like?

Fix a finite time horizon $T > 0$, and consider a sufficiently *nice* map

$$f : [0, T] \to [0, \infty)$$

that describes intensity of rainfall. Then for $t > 0$ and drainage profile g we are interested in what function f causes

$$\int_0^t f(s)\, \dot{g}(t-s)\, ds$$

to be large, which corresponds to a strong river flow at time t.

We are interested in studying the relationship between rainfall, drainage profile, flow rate, and hope this information can be used in predicting and preventing disastrous flood events.

When do Floods happen?

We want to understand whether floods for a certain drainage profile are caused by:

- Sharp but extremely unlikely rainfall
- Compounding of shallower but still significant rainfall
- Somewhere in the middle of the two above

This relates to an area of probability called *large deviation theory*, which gives a rigorous footing to the study of *rare* events.

Problem

Let $\mathcal{R}$ be the family of rainfall mappings in the top 1% in terms of maximising flow. What do rainfall mappings in $\mathcal{R}$ look like?

Small Catchment Similarity

If we can define a *good* notion of *distance* between the trees of two catchments then we have a better understanding of how tree structure is related to river flow.

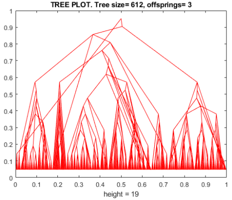
Idea (Use pointed Gromov-Hausdorff-Prokhorov distance)

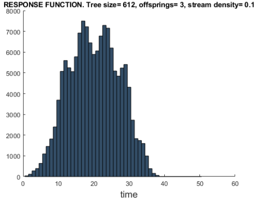
The mapping from a tree to its drainage profile is continuous with respect to GHP, so similar trees should have similar drainage profiles.

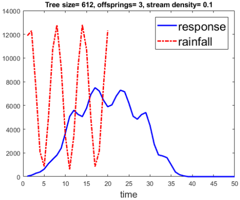
Problems

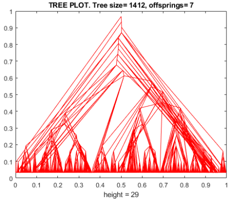
How well is this similarity captured by GHP? Is there a weaker metric we can use instead? Is there a better notion of distance?

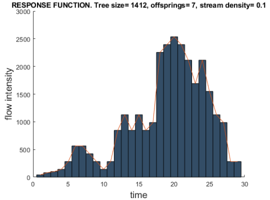
Examples with Flow Plots

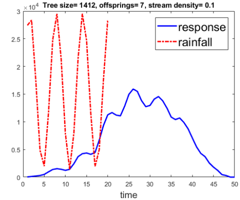












What Happens Next

1. Find a simple way to turn knowledge of the small catchment into a tree structure
2. For a given tree, understand how extreme events are related to rainfall mappings (link with existing research on rainfall)
3. Rigorously define a notion of behavioural similarity between trees
4. Extend the theory to trees which evolve over time (capture time to peak flow better)
5. Interpret how any findings can be used practically