Physical Models for Nitrate Pollution

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The Problem

Problem:
Create a physical basis that will allow us to take a more informed statistical approach in analysing data.

Aims:

- To better understand how surface pollution contributes to borehole measurements
- To understand link between borehole measurements over time
- To create a physical model that can be used to improve the current statistical approach

Background

- Large amounts of data
- Difficult to infer underlying processes
- Model parameters vary widely
- Geometry is complex

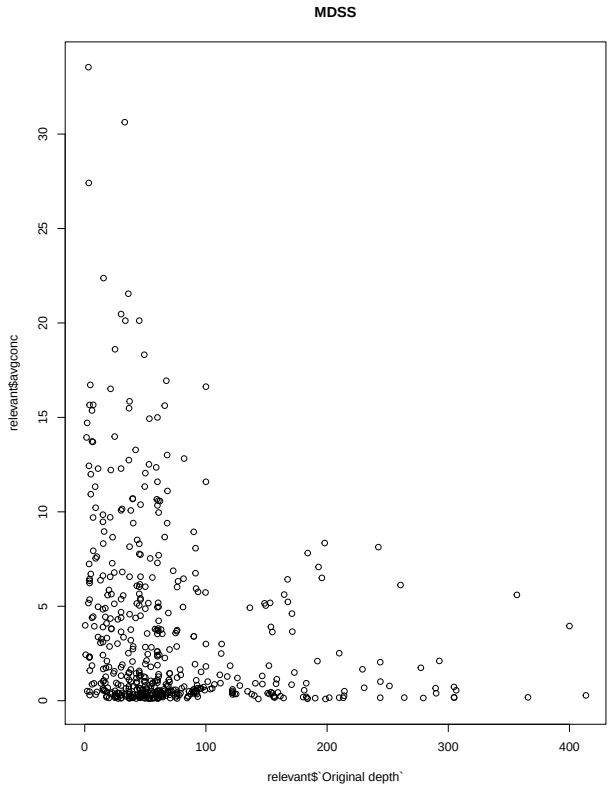

Figure: Depth vs Avg Concentration: Mean = 3.53, Sd = 4.83.

Physical Basis

Advection-Diffusion Equation: $\boxed{c_t + u \cdot \nabla c = D\nabla^2 c}$

- 2 timescales:
  - Diffusive timescale: $\frac{L^2}{D} \sim \mathcal{O}(100,000 years)$
  - Advective timescale: $\frac{L}{k} \sim \mathcal{O}(1 - 10 years)$
  - Diffusive timescale much longer
- First approximation: ignore diffusive term
  - Transport Equation
  - Travelling wave solution $c = f(z - At)$

Initial Computations – Horizontal 1D Model

Governing Equations:

- Darcy's Law: $u = -kh_x$
- Mass Conservation: $h_t = -(uh)_x + S_w$
- Advection-Diffusion of Nitrates: $c_t + uc_x = Dc_{xx} + S_c$

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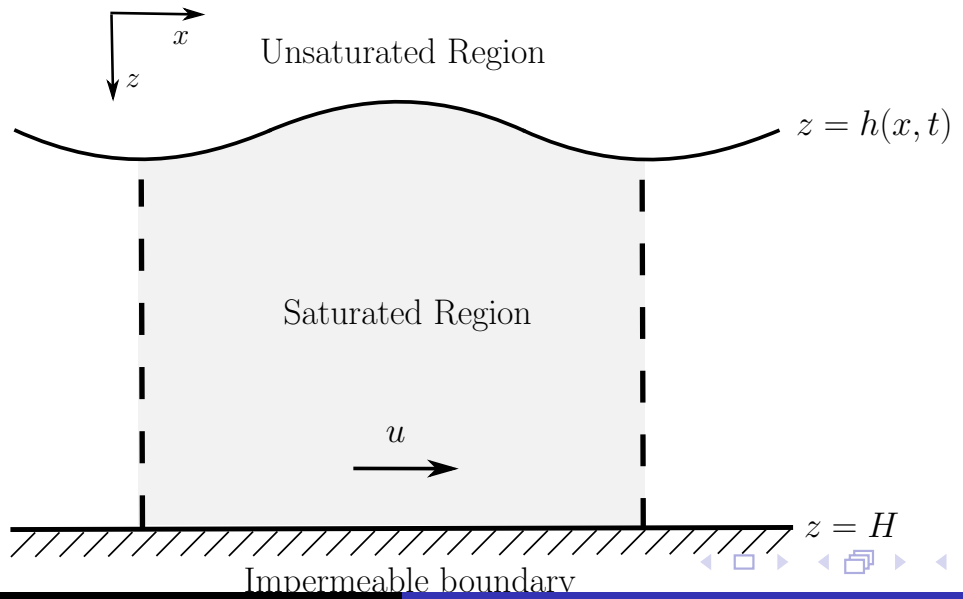
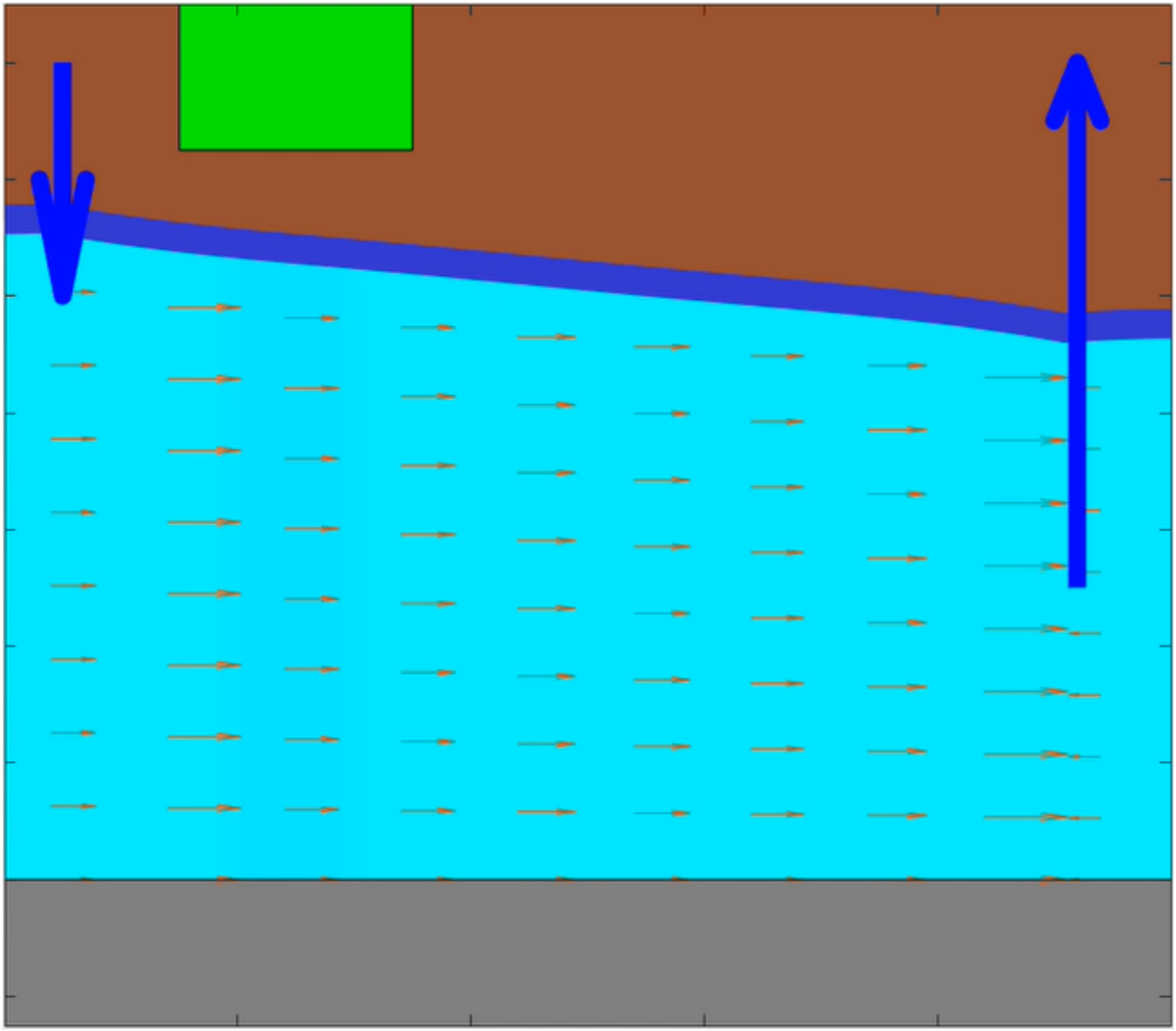


Boundary Conditions

- Impermeable bedrock
- Inlet of clean water to left
- Borehole extracting water at the right
- Source of pollutant from above

Numerical Solution



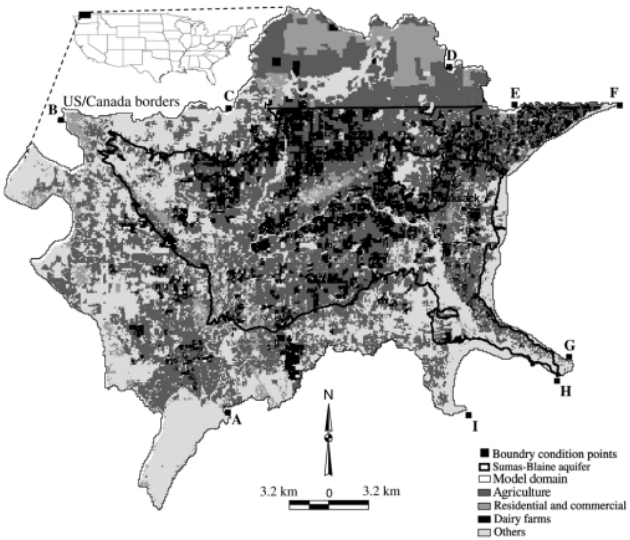
Industrial Uses

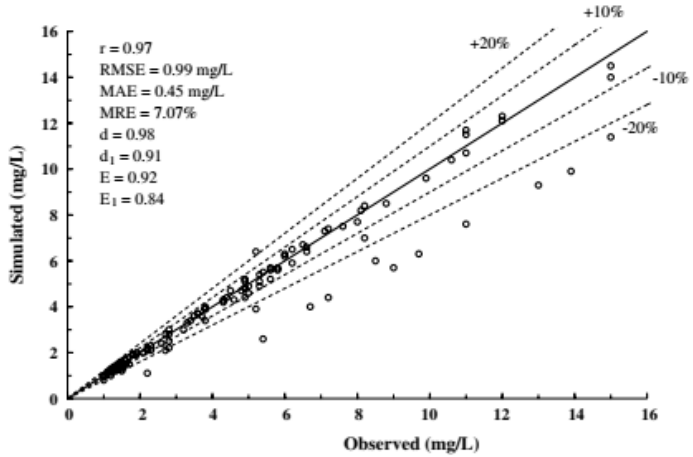
- Link surface contamination to borehole measurements
- Agricultural load as source term
- Determine causal link to concentration changes

- Predict travel-time between boreholes
- Time lag for correlations between measurements
- Improves "distance" metric for Kriging process (see Malena Sabate's talk)

Direction of Further Work

- Add more complexity to model (2D, geometry, varying properties, unsaturated zone)
- Possible to incorporate other effects (reaction rate, pumping)
- Investigate how the data can inform parameters for these effects
- Stochastic modelling of PDE coefficients, geometry and BCs to quantify uncertainties





$$R\left(\theta c\right)_t = \left(D_{ij} c_{x_j}\right)_{x_i} - \left(v_i c\right)_{x_i} + \frac{q_s S_c}{\theta} - \lambda\left(c + \frac{\rho_b \bar{c}}{\theta}\right)$$

Almasri, M. and Kaluarachchi, J. J., "Integrated modeling of nitrate contamination of groundwater in agriculture-dominated watersheds" (2007). CEE Faculty Publications. Paper 1294.

Network

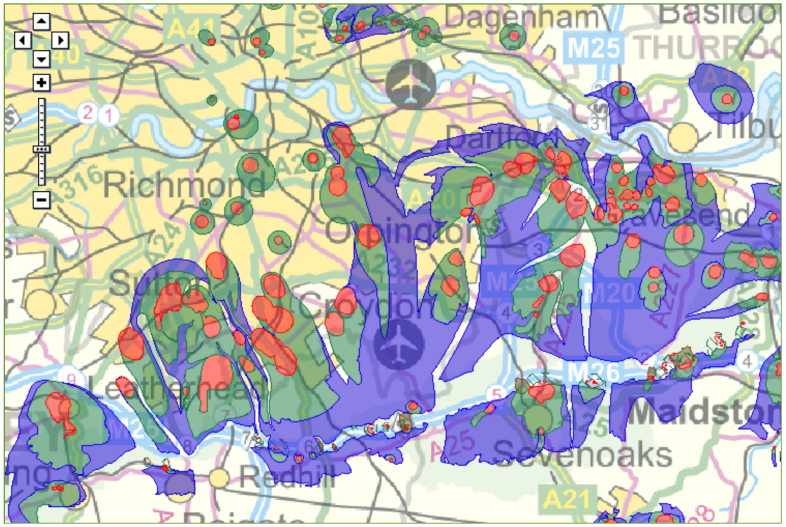


Figure: Travel-time heatmap. Source: EA Website

- Given a point (x, y), find nearest source (x_0, y_0).
- Travel time $T(x, y; x_0, y_0)$: relate to "distance" metric.