Source Identification and Tracking

NPL Problem

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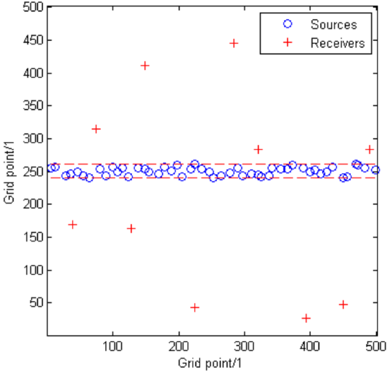
Problem formulations



- Tracking: Kalman Filter
- Identification: Elastic Net - modify Lasso regression

Extented Kalman Filter

- State vector: $x(t) = [x_t, y_t, v_{x_t}, v_{y_t}, I_t]^T$

  Measurement vector: $z(t) = [I_1, I_2, ..., I_n]^T$

- State equation: $x(t+1) = F(t)x(t) + w(t), t = 0, 1, ...$

  Measurement equation: $z(t) = h(x(t)) + v(t), t = 0, 1, ...$

- j^{th} sensor: $I_j = \sum_k c_1 \exp(-c_2 \sqrt{(x_k - u_j)^2 + (y_k - v_j)^2})$

Extented Kalman Filter

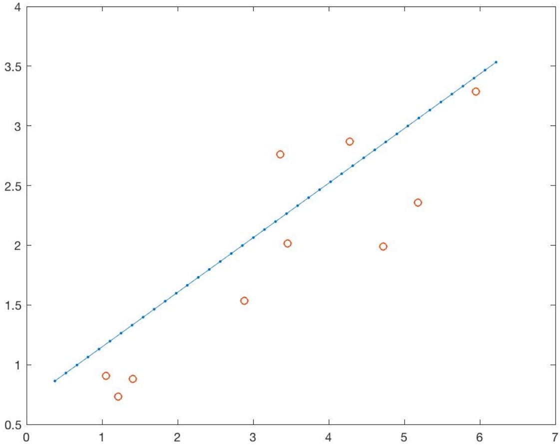


Figure: one ship, 10 sensors

Extented Kalman Filter

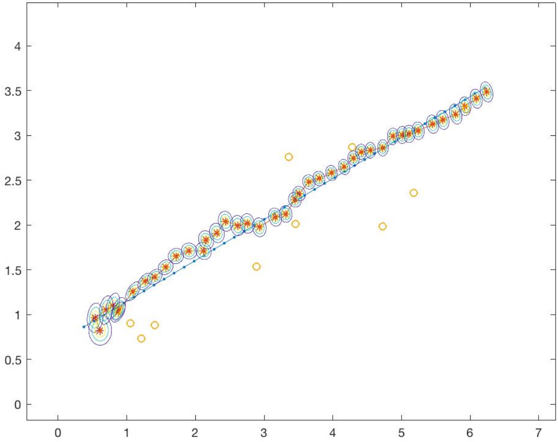


Figure: one ship, 10 sensors, low-level noise added to initial state

Extented Kalman Filter

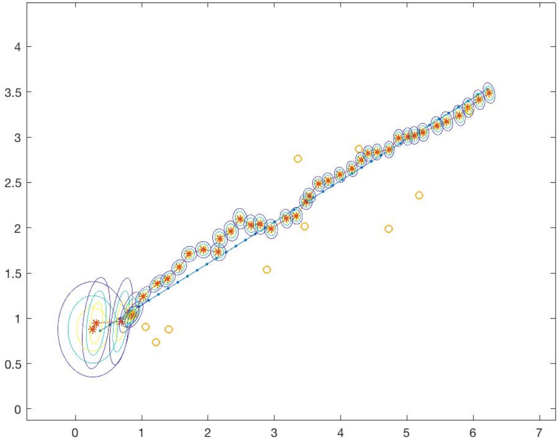


Figure: one ship, more noise in the initial state

Extended Kalman Filter - Initial Result

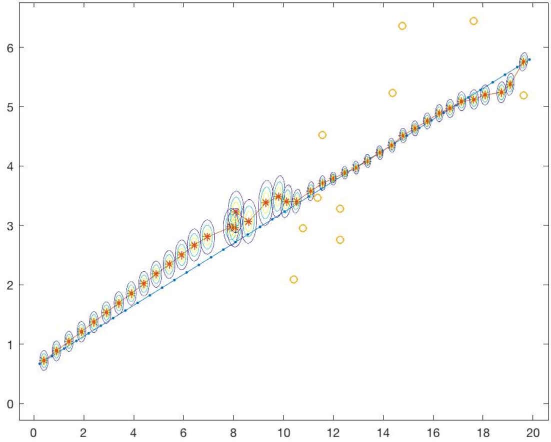


Figure: Noise level 0.1 in Measurements

Extended Kalman Filter - Initial Result

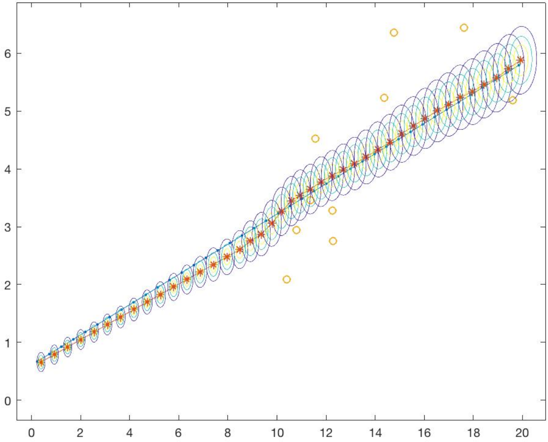


Figure: Noise level 0.5 in Measurements

Future work

- better algorithm
- physical model of sound propagation
- more than one ship

Elastic Net

Use location data as additional penalty term in the LASSO-optimisation

$$\|As - y\|_2^2 + \tau\|s\|_1 + \alpha\|s - s_0\|_2^2$$

where

$$s_0 = \begin{cases} \text{expected noise level of a ship} & \text{known ship location} \\ 0 & \text{otherwise} \end{cases}$$

Problem

- The l_2 norm works against the l_1 norm, making the result less sparse.

Elastic Net

- Only add penalty for known ship locations

$$\|As - y\|_2^2 + \tau\|s\|_1 + \alpha \sum_{k=1}^{l} \|s_{i_k} - s_0\|_2^2$$

  where $i_1, \ldots, i_l$ are the ships with known positions.
- Example: Run the provided 1d-simulation with high noise.
  - $s_0 \equiv 1$
  - $\sigma = 0.07$
  - $\alpha = 0.05$ (optimised by hand)
  - 3 ships with known locations, 5 overall

Elastic Net - Initial Result