Uncertainty in Acoustic Sensor Networks

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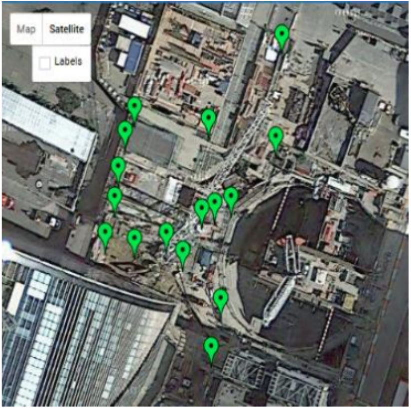
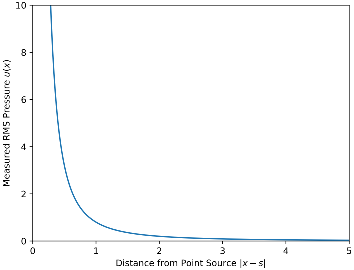
The Problem



- 16 Receivers
- 26 Frequencies
- 1 Goal: Reconstruct acoustic pressure everywhere, with uncertainty

Our Idea: Pseudo-Sources

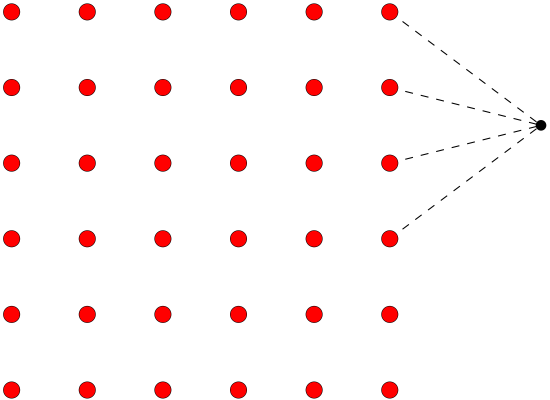


Pressure due to point source at $\mathbf{s}$ with amplitude A is

$$u(\mathbf{x}) \propto \frac{A}{\omega \|\mathbf{x} - \mathbf{s}\|}$$

Idea: Describe pressure field in terms of point sources.

Pseudo-Source Lattice



$$u_j(\mathbf{x}) = \sum_{i=1}^{S} \frac{A_{ij}}{\omega_j \|\mathbf{x} - \mathbf{s}_i\|}$$

Pseudo source i emits on frequency ω_j with amplitude A_{ij}
Observe (noisy) pressure Y_{kj} on frequency ω_j
at fixed location $\mathbf{x} = \mathbf{r}_k$

Bayesian Inference using Gaussian "Bodge"

After reshaping A_{ij} and Y_{kj} into vectors $\mathbf{c}$ and $\mathbf{y}$;

Prior $$\mathbf{c} \sim N(\boldsymbol{\mu}, \sigma_a^2 I)$$
Likelihood $$\mathbf{y}|\mathbf{c} \sim N(D\mathbf{c}, \sigma^2 I)$$

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Posterior:

$$\mathbf{c}|\mathbf{y} \sim N(\boldsymbol{\mu}', \Sigma)$$

where

$$\Sigma = \left(\frac{1}{\sigma^2} D^T D + \frac{1}{{\sigma_a}^2} I\right)^{-1}$$

$$\boldsymbol{\mu}' = \Sigma\left(\frac{1}{\sigma^2} D^T \mathbf{y} + \frac{1}{{\sigma_a}^2} \boldsymbol{\mu}\right)$$

Interpretation as a Gaussian Process

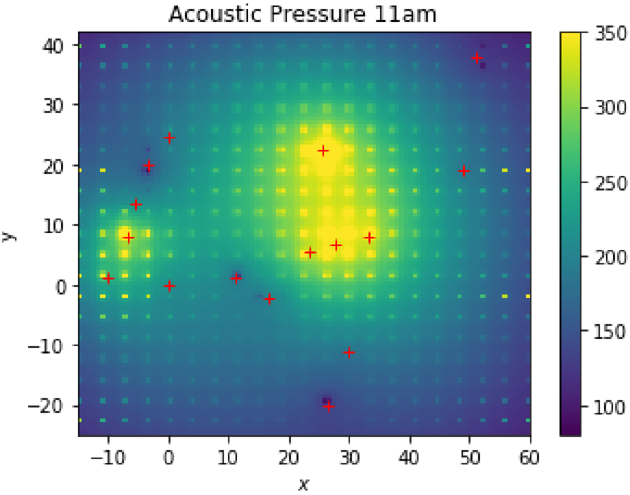
Mean function for ω_j field:

$$\mu_j(\mathbf{x}) = \frac{1}{\omega_j} \sum_{i=1}^{S} \frac{\mu'_{ij}}{\|\mathbf{x} - \mathbf{s}_i\|}$$

Kernel for ω_j field:

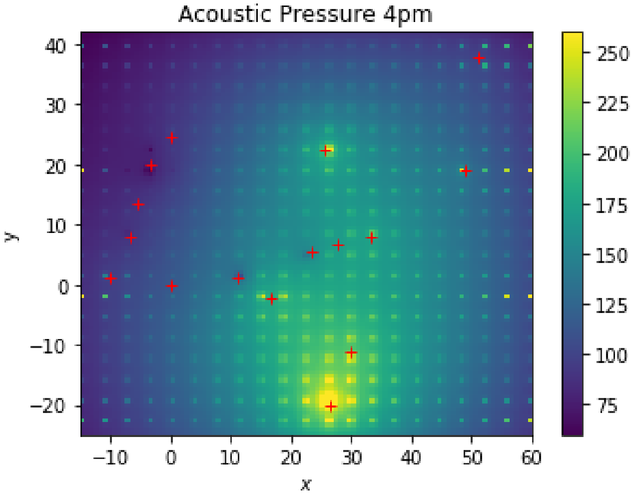
$$k_j(\mathbf{x}, \mathbf{x}') = \frac{1}{{\omega_j}^2} \sum_{i=1}^{S} \sum_{k=1}^{S} \frac{\mathrm{Cov}(A_{ij}, A_{kj})}{\|\mathbf{x} - \mathbf{s}_i\| \|\mathbf{x}' - \mathbf{s}_k\|}$$

Results



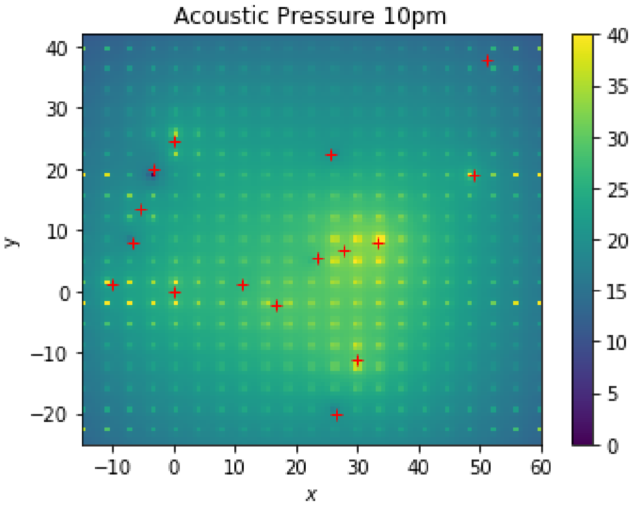
Acoustic Pressure 11am
40
30
20
10
0
−10
−20
y
−10
0
10
20
30
40
50
60
x
350
300
250
200
150
100

Results



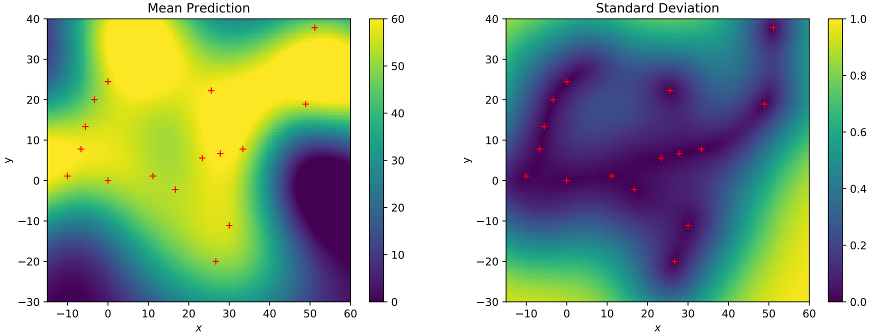
Acoustic Pressure 4pm
y
x
−10
0
10
20
30
40
50
60
−20
250
225
200
175
150
125
100
75

Results



Acoustic Pressure 10pm
y
x
40
30
20
10
0
−10
−20
−10
0
10
20
30
40
50
60
40
35
30
25
20
15
10
5
0

Squared-Exponential Kernel (No Physics!)



Onwards

Now:

- Improvement on standard GP
- Includes some physics
- Not completely physical

Future:

- Completely physical representation
  - Lose Gaussian process structure
  - Spatial Statistics?
- Incorporate environment
  - Modelling, PDEs

Thanks!