Aquifer as Random Environment

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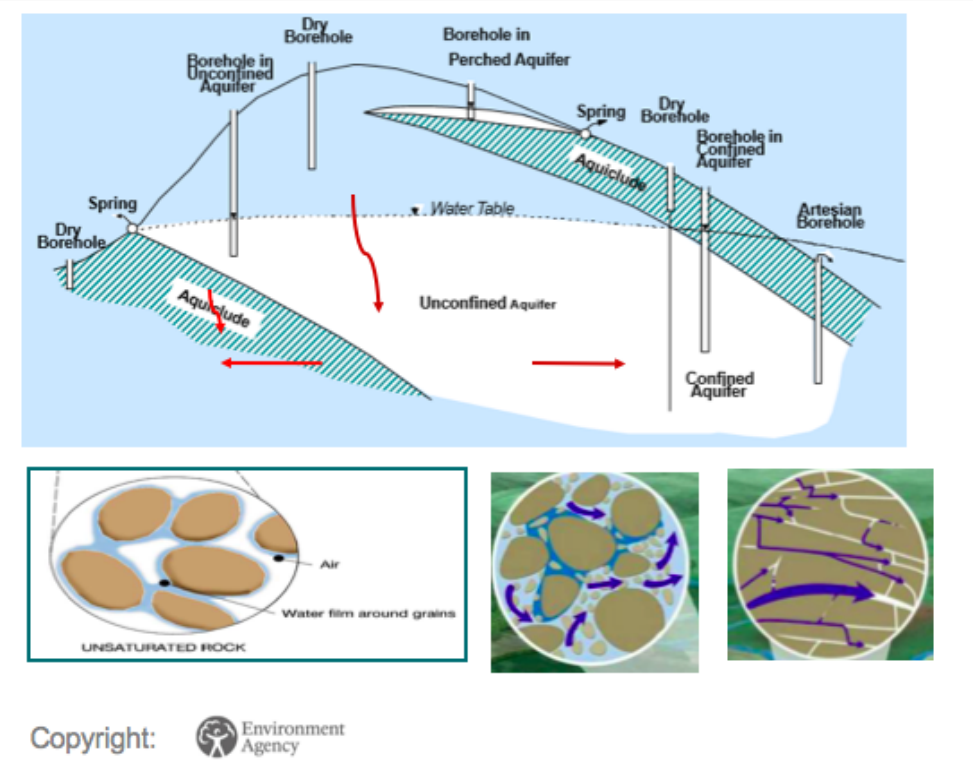
ITT7

Motivation.

Modelling flow of polluted water through porous medium $\longrightarrow$ inhomogeneous structure.



Consider models with only a few parameters.

Recall: Questions for the week.

- Look at one dimensional toy example: to understand transport of pollutant through water flow on a line.
- Simplified equation for concentration of pollutant:

$$\frac{\partial C}{\partial t} = -v\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2}.$$

- What is a good model for random fields v, D?

The Environment Diffusivity

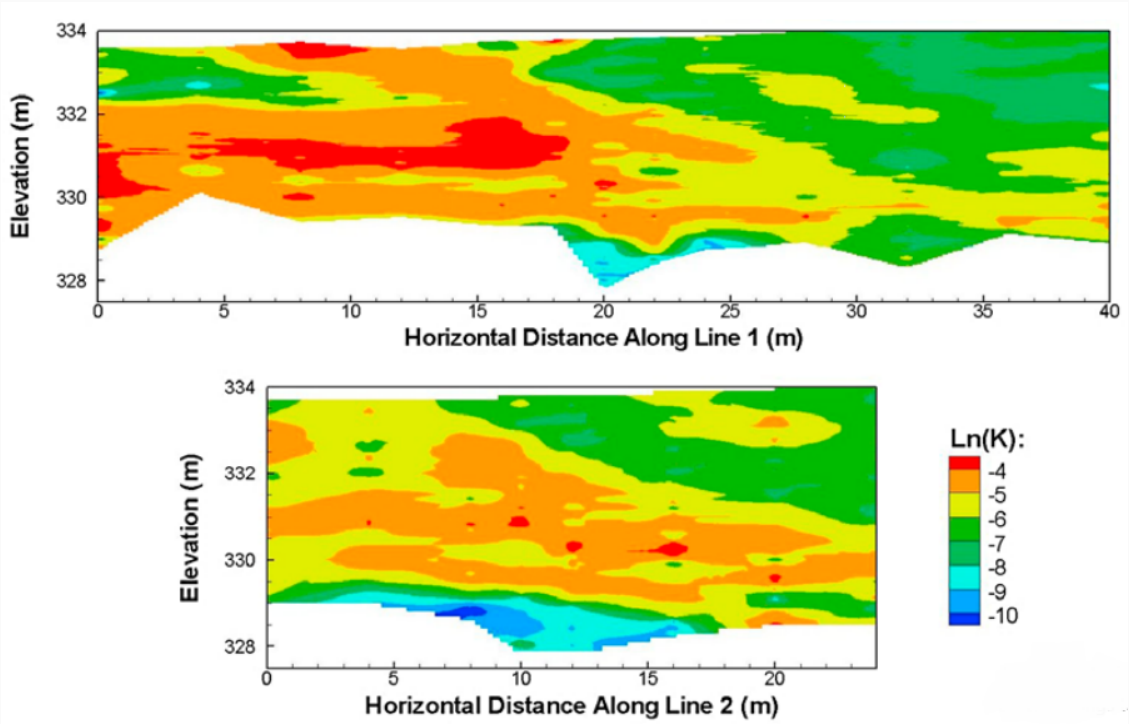
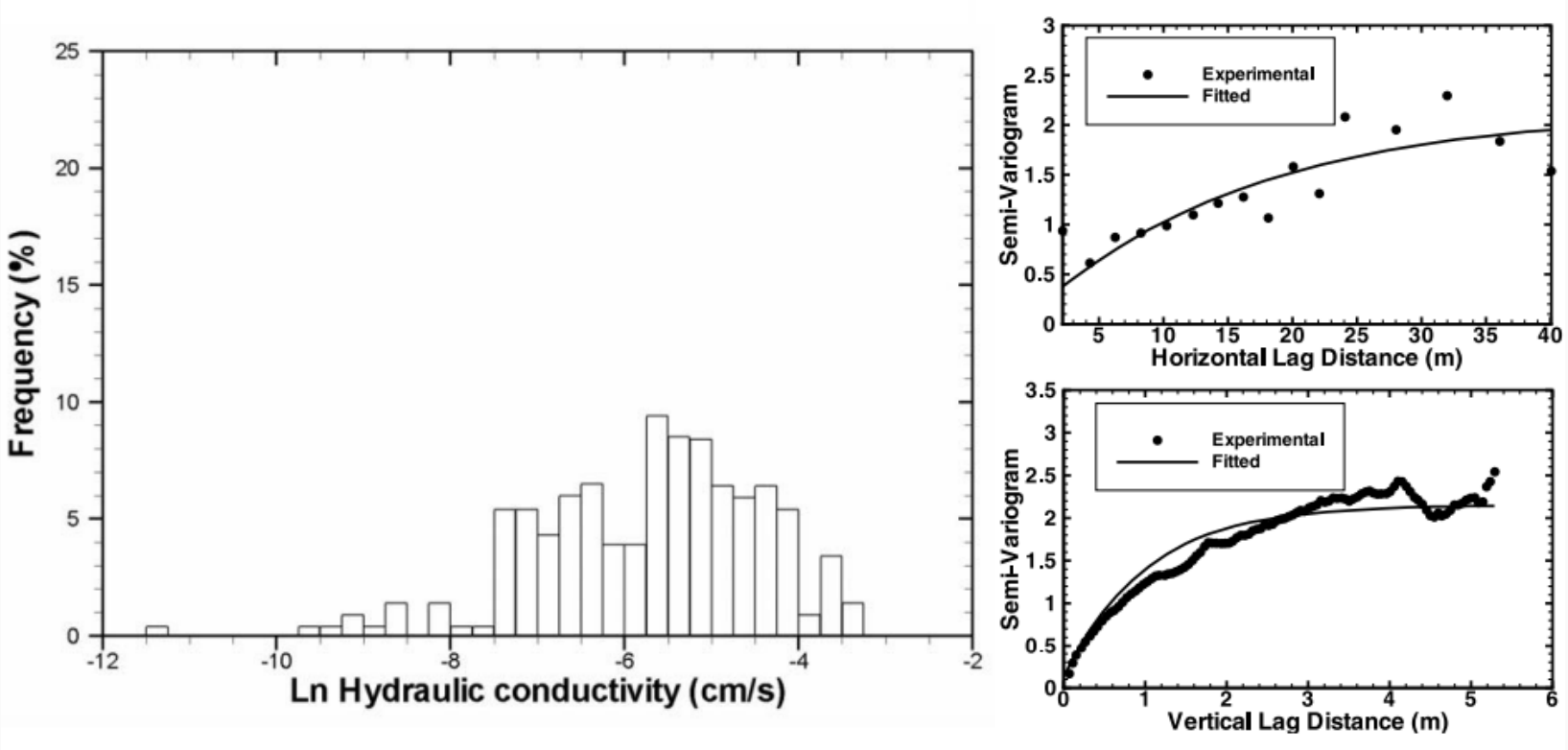


Figure 3. The spatial variability of hydraulic conductivity along (a) line 1 and (b) line 2.

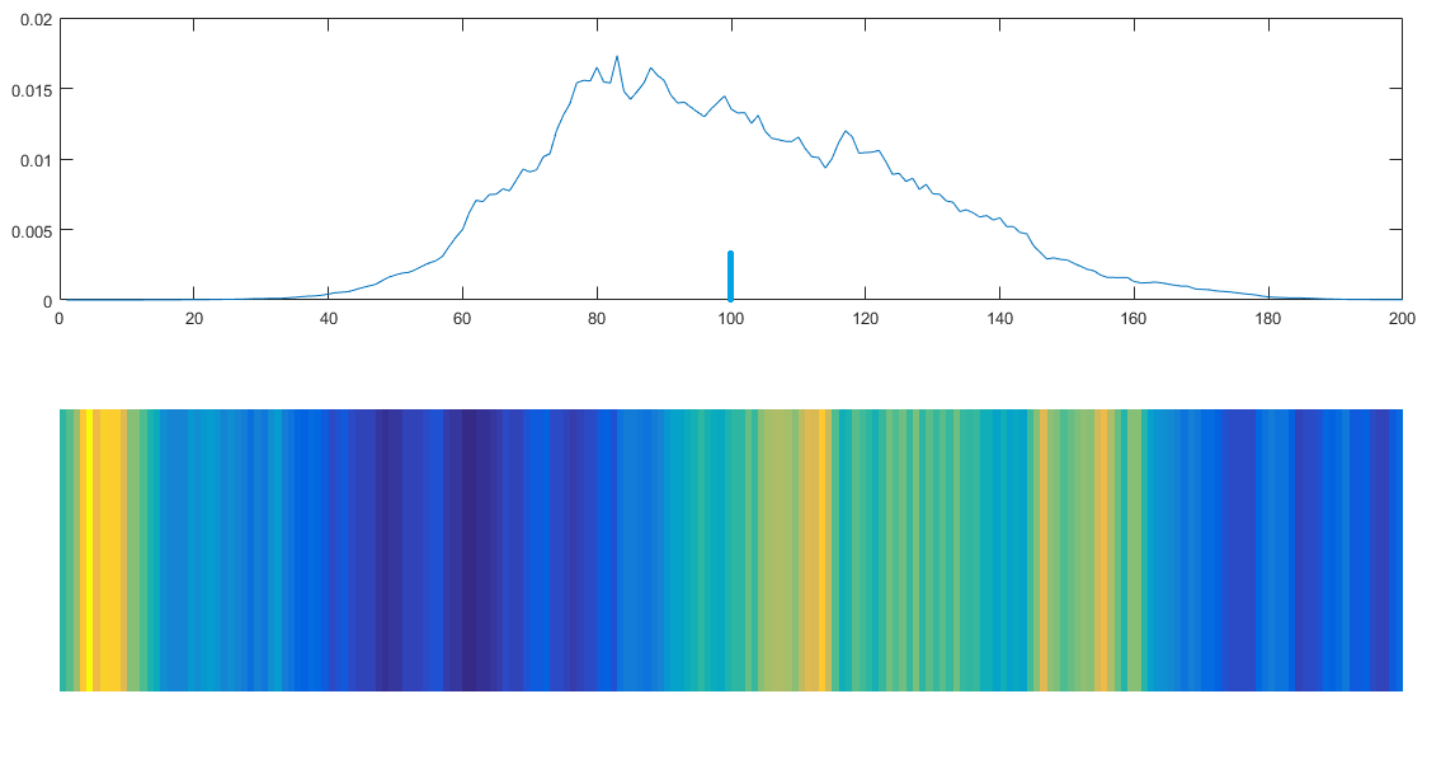
Sudicky et al. (2010) investigate diffusivity in a shallow unconfined aquifer located at North Bay, Ontario. They observe a *log-Gaussian random field*.



Woodbury/Ulrych (2000) suggest an exponential covariance structure and above Sudicky et al. (2010) estimate the covariance structure.

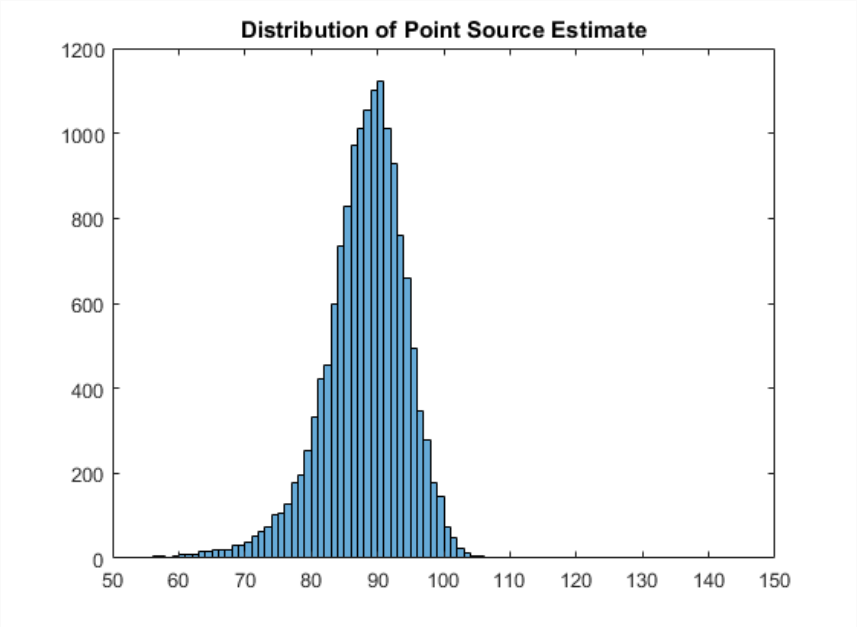
The Forwards Model

We put a flow on this environment from some point source of nitrate from some known point over some known timescale.



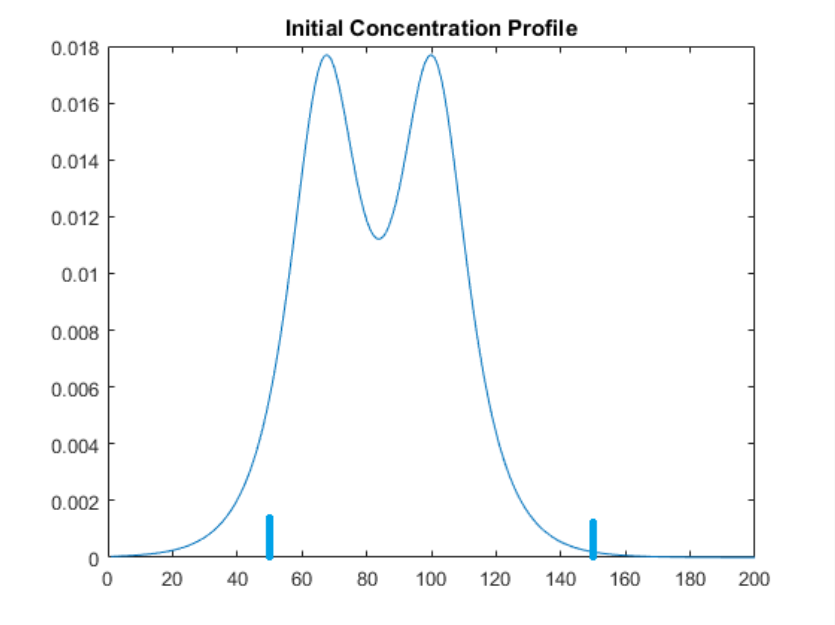
Solving the Inverse Problem

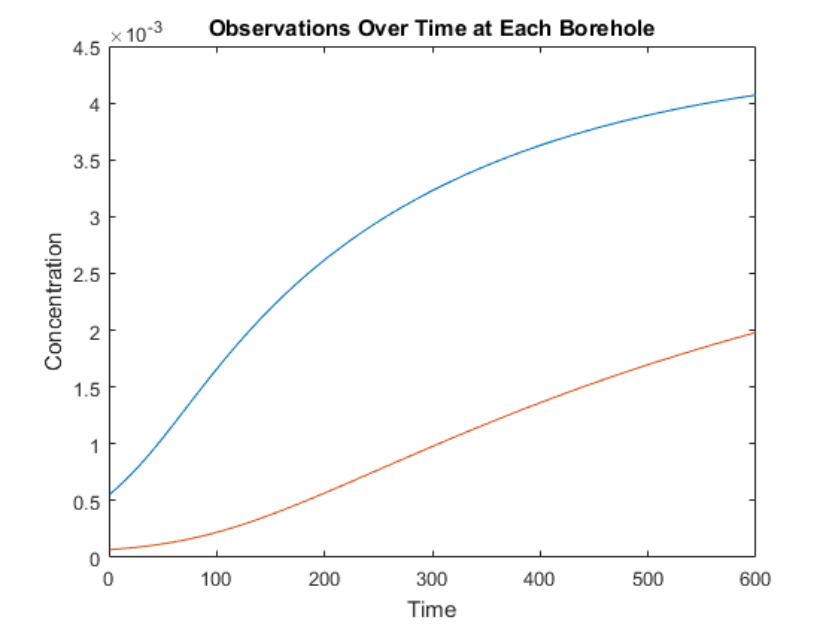
- Then given the relative intensities at our two boreholes we can estimate the position of the point source.
- Thus we have a measure of the additional uncertainty created by the unknown aquifer structure.



Extension: Data Over Time

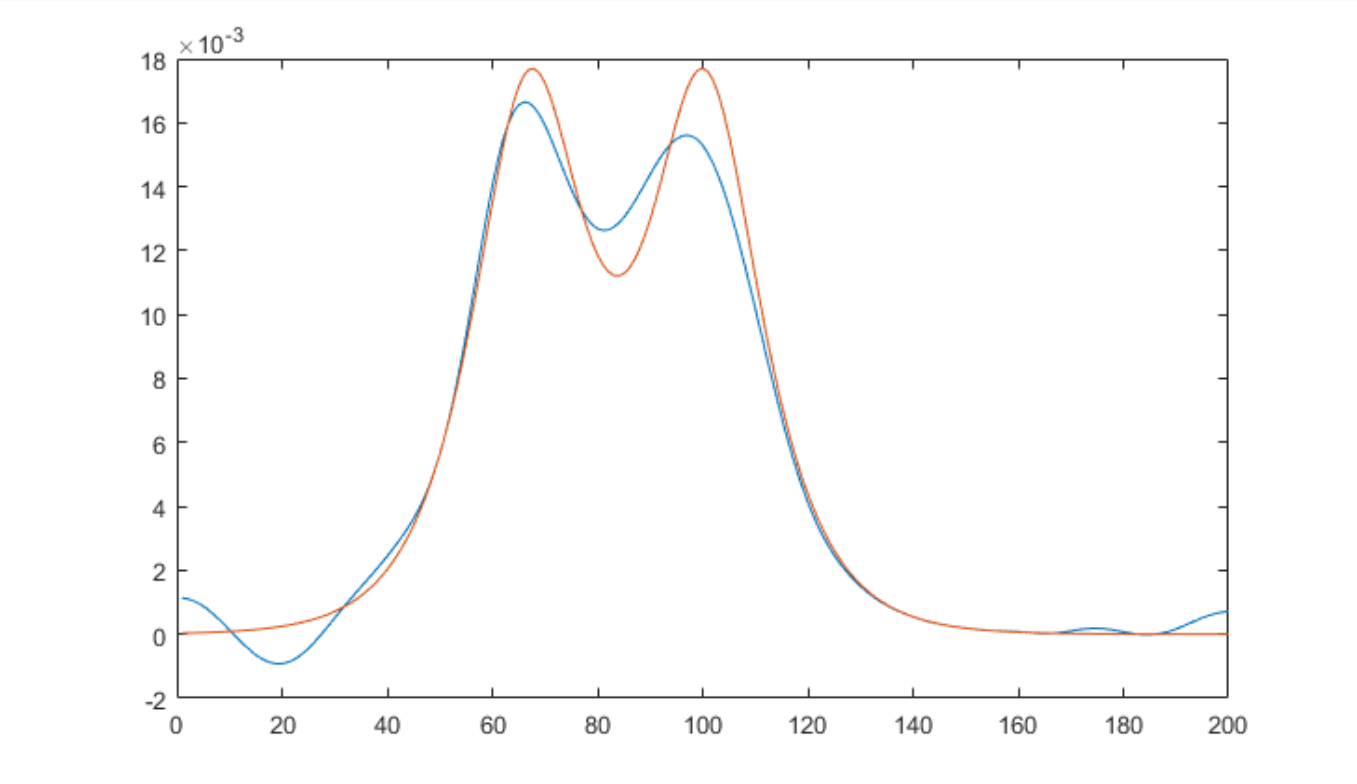
Given observations of nitrate concentration over time in two boreholes, we can try to recover the whole initial concentration profile.





Extension: Data Over Time

We achieve this by regularising for smoothness (Tikhonov). The random environment is *unknown* so we simulate an ensemble of environments and average the resultant ensemble of regularised inverses.



Further Work

- Robustness to spatial correlation scale.
- Inference of random environment?
- Putting convection back into flow with *physical* random properties.
- Rigorous results for the PDE in random environment.