NPL-SAMBA ITT projects

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Outline

Quality engineering

- Functional requirement of a manufactured part
- Manufactured parts specified in terms of geometric shape, dimensions and tolerances in computer-aid design (CAD) drawings/files
- All designs usually refer to the ideal shape at 20 ^{0}C, the reference temperature for measurement
- Manufactured parts the output of a number of cutting, machining, drilling, polishing processes
- Each process likely to drift due to tool wear, environmental effects (especially temperature)
- Other sources of variation: tool, machine, operator

Connecting rod from an internal combustion engine

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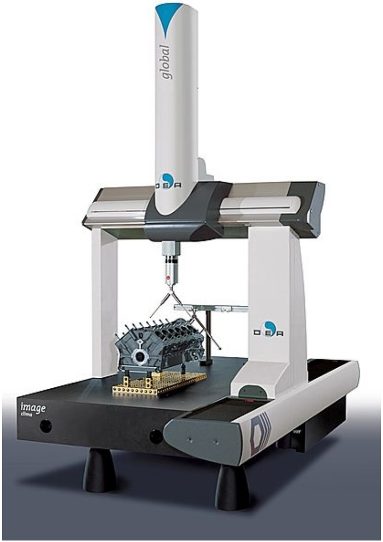
Traditional approach to inspection

- Assess a sample of objects from the production line to check dimensions and tolerances
- Plug gauges, ring gauges, hard gauges based on artefacts (yes/no test)
- Coordinate measuring machines (CMMs): gather x-, y- and z-coordinates of points $\boldsymbol{x}_i$ on the workpiece.
- Apply algorithms to $X = \{\boldsymbol{x}_i\}$ to check if the part (as presented by X) conforms to specification
- Require workpiece to reach stable equilibrium at 20 ^{0}C
- Equilibrated workpiece: use a temperature measurement to scale back to 20 ^{0}C

Coordinate measuring machine



In-process measurement

Measure the workpiece , safe time, money

- Workpiece ideal geometry at 20 ^{0}C specified, with tolerances
- Workpiece being manufactured: cutting, drilling, machining, cooling
- Measurements of the temperature at finite number of locations on the workpiece
- Measurements of the dimensions of a finite number of key features
- Using a FE model of artefact and the measurements, infer the workpiece shape at an equilibrated 20 ^{0}C

Data assimilation

Major tool in weather prediction

- Forward model: given the ideal geometry at 20 ^{0}C, known material properties, known boundary conditions, predict the geometric distortion due to thermal effects
- Inverse problem: incomplete, measured boundary conditions, approximate values of material properties
- Uncertainty quantification: how well can we infer the shape at 20 ^{0}C
- Degrees of freedom/surrogate models/machine learning: what measurement information is sufficient to make good inferences
- Bing Ru Yang, Bath, Louise Wright, Dale Partridge, NPL, project work

Large engineering structures

- Aircraft wings, bridges
- Industry 4.0, digital twins

Outline

Fitting a model to data

- Standard data fitting model

$$\boldsymbol{y} = C\boldsymbol{a} + \epsilon, \quad \epsilon \in \mathrm{N}(\boldsymbol{0}, \sigma^2 I)$$

- $\boldsymbol{y}$ is an $m \times n$ data vector, $\boldsymbol{a}$ parameters of the model
- C is an $m \times n$ observation matrix, e.g. basis functions evaluated at $\boldsymbol{x}$
- ϵ is an $m \times n$ vector of independent random effects associated with the measuring system
- Least squares model fit

$$\hat{\boldsymbol{a}} = (C^{\mathrm{T}}C)^{-1}C^{\mathrm{T}}\boldsymbol{y} = R_1^{-1}Q_1^{\mathrm{T}}\boldsymbol{y}, \quad C = Q_1R_1$$

$$\hat{\boldsymbol{y}} = C\hat{\boldsymbol{a}} = C(C^{\mathrm{T}}C)^{-1}C^{\mathrm{T}}\boldsymbol{y} = Q_1Q_1^{\mathrm{T}}\boldsymbol{y}$$

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Effective number of degrees of freedom in a model

- If $\hat{\boldsymbol{y}} = H\boldsymbol{y}$, the sum of the eigenvalues of H is a measure of the number of degrees of freedom associated with the model.
- Least squares model fit

$$\hat{\boldsymbol{y}} = C(C^{\mathrm{T}}C)^{-1}C^{\mathrm{T}} = Q_1 Q_1^{\mathrm{T}}\boldsymbol{y}$$

- $Q_1 Q_1^{\mathrm{T}}$ is a projection with n eigenvalues equal to 1, all others 0.

Correlated systematic effects

- Extension of the standard model:

$$\boldsymbol{y} = C\boldsymbol{a} + \boldsymbol{e} + \epsilon, \quad \boldsymbol{e} \in \mathrm{N}(\boldsymbol{0}, V_0), \quad \epsilon \in \mathrm{N}(\boldsymbol{0}, \sigma^2 I)$$

Gauss Markov regression

- Combined variance matrix, Choleski decomposition

$$V = V_0 + \sigma^2 I = LL^{\mathrm{T}}, \quad \tilde{\boldsymbol{y}} = L^{-1}\boldsymbol{y}, \quad \tilde{C} = L^{-1}C$$

$$\tilde{\boldsymbol{y}} = \tilde{C}\boldsymbol{a} + \tilde{\epsilon}, \quad \tilde{\epsilon} \in \mathrm{N}(\boldsymbol{0}, I)$$

- Effective degrees of freedom: transformed problem

$$\hat{\tilde{\boldsymbol{y}}} = \tilde{Q}_1 \tilde{Q}_1^{\mathrm{T}} \tilde{\boldsymbol{y}}$$

- Effective degrees of freedom: original problem

$$\hat{\boldsymbol{y}} = L\hat{\tilde{\boldsymbol{y}}} = L\tilde{Q}_1 \tilde{Q}_1^{\mathrm{T}} L^{-1}\boldsymbol{y}$$

Explicit effects model

- Same extended model

$$\boldsymbol{y} = C\boldsymbol{a} + \boldsymbol{e} + \epsilon, \quad \boldsymbol{e} \in \mathrm{N}(\boldsymbol{0}, V_0), \quad \epsilon \in \mathrm{N}(\boldsymbol{0}, \sigma^2 I)$$

- Introduce parameters to describe the systematic effects,

$$\boldsymbol{e} = L_0\boldsymbol{d}, \quad V_0 = L_0 L_0^{\mathrm{T}}$$

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} C & L_0 \\ & I \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{d} \end{bmatrix} + \begin{bmatrix} \epsilon \\ \delta \end{bmatrix} \quad \epsilon \in \mathrm{N}(\boldsymbol{0}, \sigma^2 I), \quad \delta \in \mathrm{N}(\boldsymbol{0}, I)$$

Augmented system

$\tilde{\boldsymbol{y}} = \tilde{C}\tilde{\boldsymbol{a}} + \tilde{\epsilon}$, where

$$\tilde{\boldsymbol{y}} = \left[\begin{array}{c} \boldsymbol{y}/\sigma \\ \boldsymbol{0} \end{array}\right], \quad \tilde{C} = \left[\begin{array}{cc} C/\sigma & L_0/\sigma \\ & I \end{array}\right]$$

and

$$\tilde{\boldsymbol{a}} = \left[\begin{array}{c} \boldsymbol{a} \\ \boldsymbol{d} \end{array}\right], \quad \tilde{\epsilon} = \left[\begin{array}{c} \epsilon \\ \delta \end{array}\right] \quad \tilde{\epsilon} \in \mathrm{N}(\boldsymbol{0}, I)$$

- Eigenvalues

$$\hat{\tilde{\boldsymbol{y}}} = P\tilde{\boldsymbol{y}} = \left[\begin{array}{cc} P_{11} & P_{12} \\ P_{21} & P_{22} \end{array}\right] \left[\begin{array}{c} \boldsymbol{y}/\sigma \\ \boldsymbol{0} \end{array}\right]$$

$$\hat{\boldsymbol{y}} = P_{11}\boldsymbol{y}$$

- $n \leq \sum_j \lambda_j(P_{11}), \ \sum_j \lambda_j(P_{22}) \leq m$

Gaussian Processes

- Same extended model

$$\boldsymbol{y} = C\boldsymbol{a} + \boldsymbol{e} + \epsilon, \quad \boldsymbol{e} \in \mathrm{N}(\boldsymbol{0}, V_0), \quad \epsilon \in \mathrm{N}(\boldsymbol{0}, \sigma^2 I)$$

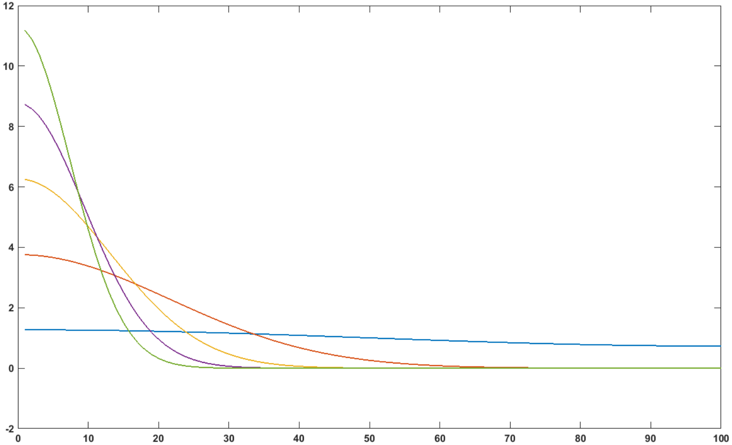
- $C_{ij} = b_j(t_i)$, $\mathrm{cov}(e, e') = k(t, t')$, e.g.

$$k(t, t') = \sigma_E^2 \exp\left\{-(t - t')^2/\tau^2\right\}$$

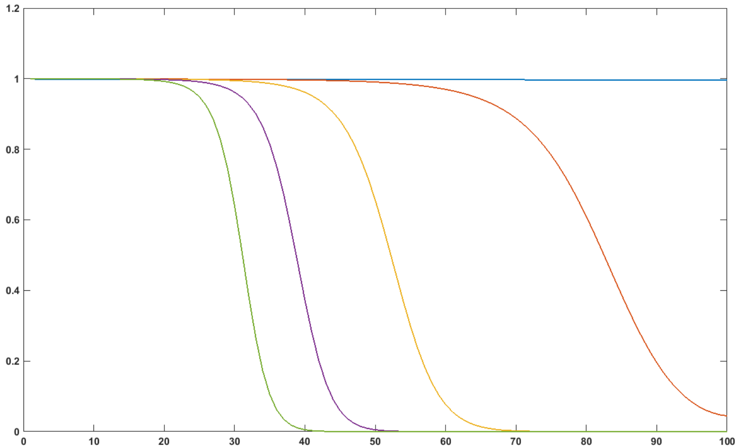
- Equally spaced t_i

$$V = \sigma_E^2 \begin{bmatrix} 1 & v & v^4 & v^9 & v^{16} & \cdots \\ v & 1 & v & v^4 & v^9 & \cdots \\ v^4 & v & 1 & v & v^4 & \cdots \\ & & & \ddots & & \end{bmatrix}$$

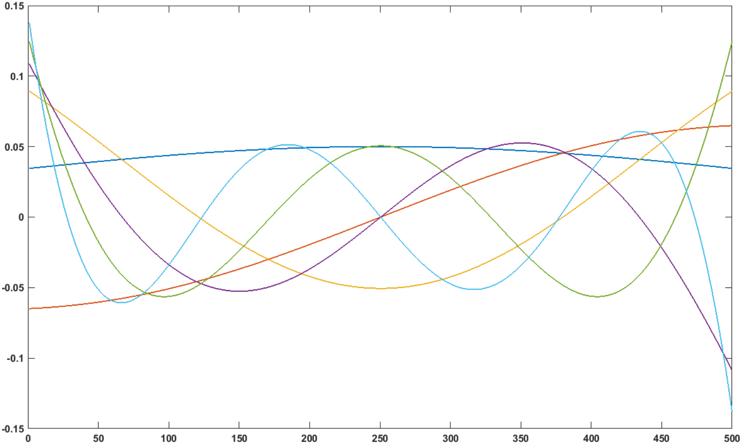
Eigenvalues of V for different τ



Eigenvalues of P_{11} for different τ



Eigenvectors of V



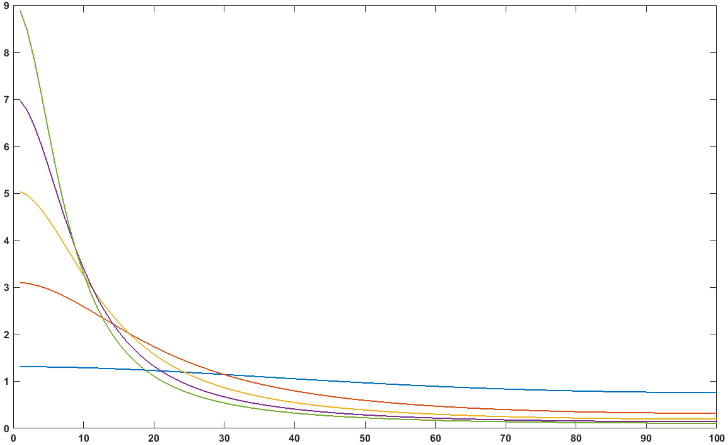
Eigenvectors as Chebyshev polynomials

0.0838	-0.0002	0.0549	0.0009	0.0400	0.0018
0.0001	0.0724	-0.0004	-0.0485	-0.0013	-0.0366
-0.0077	0.0001	0.0697	0.0007	0.0461	0.0017
-0.0000	-0.0078	0.0001	-0.0687	-0.0009	-0.0449
0.0003	-0.0000	-0.0079	-0.0001	0.0681	0.0011
0.0000	0.0004	-0.0000	0.0080	0.0002	-0.0677
-0.0000	0.0000	0.0004	0.0000	-0.0081	-0.0002
-0.0000	-0.0000	0.0000	-0.0005	-0.0000	0.0081
0.0000	-0.0000	-0.0000	-0.0000	0.0005	0.0000
0.0000	0.0000	-0.0000	0.0000	0.0000	-0.0005

Chebyshev polynomials as eigenvectors

11.1174	0.0445	-8.8230	0.0275	-0.6102	0.0337
-0.0000	12.8329	0.1027	-9.1725	0.0586	-0.9364
1.1822	0.0047	12.3875	0.1628	-9.1967	0.0874
-0.0000	-1.4056	-0.0112	-12.5111	-0.2227	9.1655
0.0785	0.0003	1.4235	0.0180	12.6002	0.2820
-0.0000	-0.0869	-0.0007	-1.4731	-0.0250	-12.6561
0.0034	0.0000	0.0874	0.0011	1.5080	0.0320
0.0000	-0.0036	-0.0000	-0.0900	-0.0015	-1.5330
0.0001	0.0000	0.0036	0.0000	0.0920	0.0019
0.0000	-0.0001	-0.0000	-0.0037	-0.0001	-0.0935

Eigenvalues of V, $k(t,t') \propto \exp\{-|t-t'|/\tau\}$



Eigenvectors of V, $k(t,t') \propto \exp\{-|t-t'|/\tau\}$