ITT 7: Spectral analysis associated with Gaussian Processes

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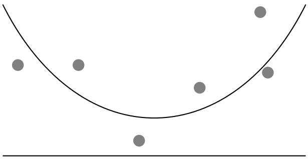
Setup

- Given: data points over time $y_i(t_i)$
- Seeking: model that fits the data
  - Problem: balance between
    - few parameter $\rightarrow$ bad fit
    - many parameters $\rightarrow$ computationally expansive/ overfitting
- Assumption: correlation between data points given by a correlation matrix/ kernel, e.g.

$$cov(e, e') = \sigma^2 \exp\{-(t - t')^2/\lambda^2\}$$

- λ large: 3 degrees of freedom
- λ small: 6 degrees of freedom



Conclusion

Eigenvalues tell us something about the effective degree of freedom of the system.

Problem formulation

- Model

$$\boldsymbol{y} = C\boldsymbol{a} + \boldsymbol{e} + \varepsilon, \quad \boldsymbol{e} \in N(\mathbf{0}, V_0), \quad \varepsilon \in N(\mathbf{0}, \sigma^2 I)$$

- for given V_0, e.g.
  - squared exponential kernel
  - exponential kernel
  - Matérn covariance function
  - any covariance matrix
- For given covariance matrix/ kernel, how do we get information about the effective degrees of freedom of a system, **in an computationally efficient way**?

Our approach

- Literature:
  - Paper by Rob:
    - enlarge covariance matrix to obtain "nicer" properties
    - only considered Matérn covariance function
  - (low rank) matrix approximation
  - Kalman filter: update the model function in real time