Detecting non-stationarity in peak river flows

Emiko Dupont, Robbie Peck, Tom Smith,
Ilaria Prosdocimi, Nicole Augustin

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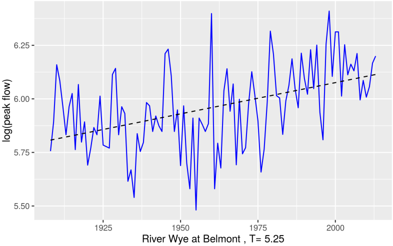
Linear model at each station



Model:

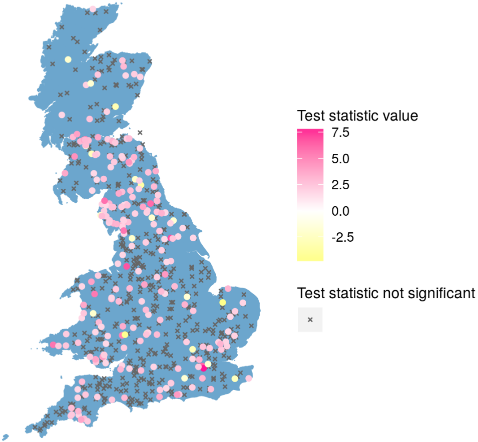
$$y_i = \log(\text{flow}_i) = \beta_0 + \beta_1 t_i + \varepsilon_i \quad \text{where} \quad \varepsilon_i \sim N(0, \sigma^2) \text{ iid}$$

Hypothesis test: $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$

$$T = \frac{\hat{\beta}_1}{\hat{\sigma}_{\hat{\beta}_1}} \sim_{\text{approx}} N(0,1) \text{ under } H_0$$

Reject H_0 if $|T| > 1.64$ (time dependence is significant)

Test statistic for $\alpha = 0.1$



Time dependence significant at around 20% of stations

What can we do to improve this?

Improvement 1: Build a spatial model for the test statistic

- Pooling data improves **power** of statistical test
- Smoothing of extreme test statistics

Approach 1: Mixed effects model based on hydrometric areas:
For station i in hydrometric area j

$$T_i = \mu + b_j + \varepsilon_i \quad \text{where} \quad b_j \sim N(0, \sigma_b^2), \quad \varepsilon_i \sim N(0, 1)$$

Approach 2: GAM with spatial random effect

$$T_i = \mu + f(\text{lon}_i, \text{lat}_i) + \varepsilon_i \quad \text{where} \quad \varepsilon_i \sim N(0, 1)$$

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Improvement 2: Investigate shape of time dependence (GAMM)
For station i at time t_k

$$\begin{aligned} log(\text{flow}_{ik}) &= f(t_k) + b_{0,i} + b_{1,i} t_k + \varepsilon_i, \\ & \quad b_{0,i} \sim N(0, \sigma_{b_0}^2), b_{1,i} \sim N(0, \sigma_{b_1}^2), \varepsilon_i \sim N(0, \sigma^2) \end{aligned}$$