Fragmentation-Coalescence Models for Bubble Dynamics

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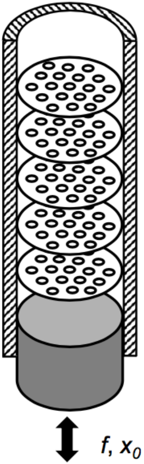
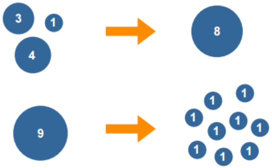
The Probabilistic Approach

- Interested in modelling the steady state bubble size distribution.
- Baffles are mixing, creating spatial homogeneity which is the main justification for non-spatial models.
- Bubbles moving fast are more likely to coalesce on contact, with fragmentation governed by the Weber number $\propto v^2 l$.



Fragmentation and coalescence

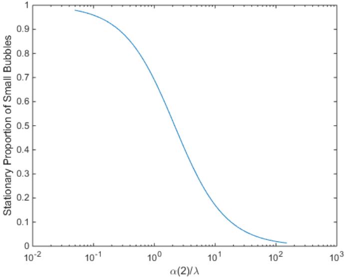


- Consider n bubbles of unit size.
- Every subset of k bubbles coalesces at rate $\alpha(k)n^{1-k}$ - for our context perhaps only $\alpha(2) \neq 0$.
- Every bubble fragments into singletons at rate $\lambda_n \to \lambda$.

Past Results

If $\omega_n(t)$ is the size of a uniformly selected bubble, with generating function G_n, then Pagett proves in his 2017 thesis that the random variable G_n converges uniformly in L^2 to a deterministic function $G(x,t)$ with

$$\frac{\partial G}{\partial t}(x,t) = \lambda(x - G(x,t)) + \sum_{k=2}^{\infty} \frac{\alpha(k)}{k!}\left(G(x,t)^k - kG(1,t)^{k-1}G(x,t)\right)$$

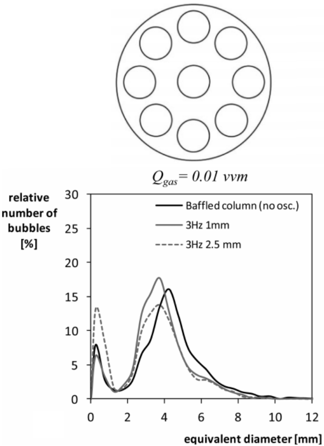


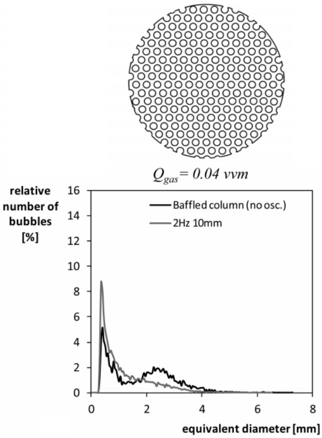
A more general model

Fragmentation was always into singletons which might lead to inaccurate predictions. Cepeda (2016) makes this model very general, with some technical conditions for well-posedness.

- A pair of bubbles of sizes x and y coalesce at rate $K(x, y)$ for some symmetric kernel K, eg. $K \equiv 1$.
- Each bubble of size x fragments at some rate $F(x)$ - the exact scaling follows from the Weber number fragementation velocity $v_c \propto x^{-1/2}$.
- For fragmentation we have some measure β on simplices which determines the relative sizes of the fragments. A suggestion for β is found in for example Sattar/Naser/Brooks (2013).

Experimental Data: Baffle Design





The resulting SDE

$$
\begin{aligned}
M(m,t) \quad = \quad & m + \int_0^t \int_{i<j} \int_0^\infty \left[c_{ij}\left(M(m,s-)\right) - M(m,s-)\right] \mathbb{1}_{\{z \le K(M_i(m,s-), M_j(m,s-))\}} \\
& \qquad \mathcal{N}(ds, d(i,j), dz) \\
& + \int_0^t \int_i \int_\Theta \int_0^\infty \left[f_{i\theta}\left(M(m,s-)\right) - M(m,s-)\right] \mathbb{1}_{\{z \le F(M_i(m,s-))\}} \\
& \qquad \mathcal{M}(ds, di, d\theta, dz)
\end{aligned}
$$

We have a Poisson-driven SDE which is another way to simulate the Markov process. The $t \to \infty$ limit, however, is not as well understood as for Pagett's model.

Looking Forward

- Is the simpler, better understood model sufficient for describing bubbles in baffled columns?
- Parameter and kernel estimation, especially a nonuniform form for K.
- Describing the $t \to \infty$ limit in Cepeda's model, with or without taking the hydrodynamic limit.

References

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