Wave inversion

Matthias, Evren, Ivan, Euan, Shaerdon, Owen, Hayley

June 9, 2017

1 dimensional problem

- Measurable
  - k (source-receiver distance)
  - t(k) (total travel time)
- Unknown
  - h (depth)
  - v (velocity)
  - θ (slope)

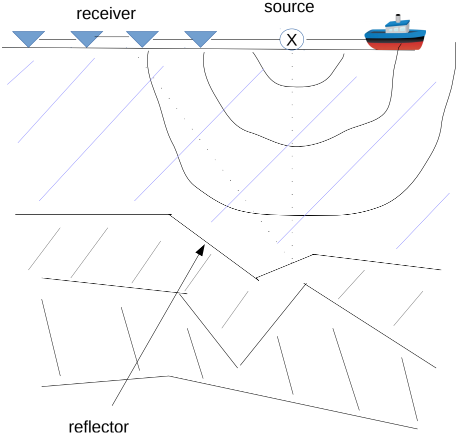

Figure: Simplified one dimensional problem

Questions at day 2 of the ITT

Q1) Given a source and a finite number of receivers, what configuration of the reflectors can you determine?

Q2) Given a fixed number of reflectors contained is some bounded area, what is the minimum number if receivers necessary to locate all the reflectors?

Q3) If we can specify that our receivers can either measure the acoustic pressure or its derivative with respect to space, how do the above answers change?

Modified questions for the ITT

1. Value of gradient measurement: Can we justify the benefit of gradient measurements?
2. What is a good basis to
   - represent the data
   - represent the image
   - to compute the wave propagation
3. Model design: What is the difference in designing the problem with respect to the data or the image?

Approach to question #1

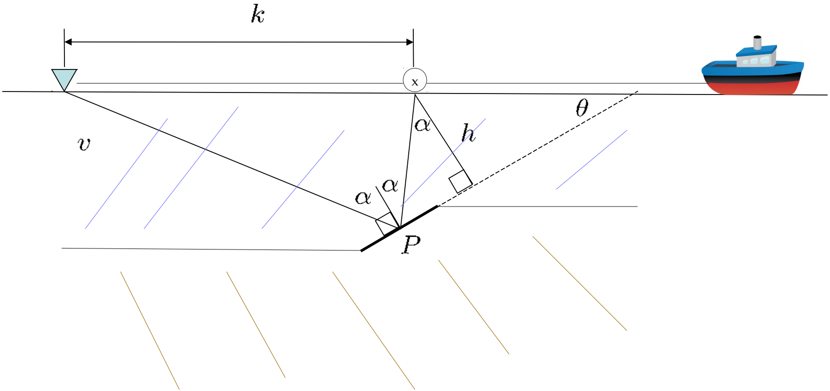

Figure: Geometry of the one dimensional problem

Approach to question #1

- Unknown: h (depth), v (velocity), θ (slope)
- We need $t(k), \partial_k t(k), \partial_k^2 t(k)$ to compute the unknowns
- Outcome of the ITT: We found a formulation to relate

$$\partial_k t(k) \sim \partial_k u$$

- Benefit: Better inversion results, where finite difference approximation fails, i.e. large type 1 sensor spacing

Figure: Left: Three type 1 sensors; Right: One type 1 and two type 2 sensors

Questions at day 5 of the ITT

1. Value of gradient measure: Can we justify the benefit of gradient measures? ✓
2. What is a good basis to
   - represent the data
   - represent the image
   - to compute the wave propagation
3. Model design: What is the difference in designing the problem with respect to the data or the image?

Approach to question #2

For an approximate solution of the wave equation

$$\frac{1}{c(x)}\frac{\partial^2 u}{\partial t^2} - \Delta u = f$$

we choose the ansatz

$$u(t,x) = aK(t-\bar{t}(x),x).$$

In order to relate

$$\nabla u\bigg|_{\partial\Omega} \sim \nabla\bar{t}\bigg|_{\partial\Omega}$$

a necessary condition for the discrete basis functions is

$$\nabla K(t-\bar{t}(x),x) = V_1(t,x)K(t-\bar{t}(x),x) - \nabla\bar{t}(x)V_2(t,x)\partial_t K(t-\bar{t}(x),x)$$

for known V_1 and V_2.

Questions at day 5 of the ITT

1. Value of gradient measure: Can we justify the benefit of gradient measures? ✓
2. What is a good basis to
   - represent the data
   - represent the image
   - to compute the wave propagation
     Necessary conditions found, basis presented satisfies this property
3. Model design: What is the difference in designing the problem with respect to the data or the image?

Approach to question #3

- Simplified wave equation

$$\frac{1}{c(x)}\frac{\partial^2 u}{\partial t^2} - \Delta u = f$$

- solution to this wave equation are called: solution of the forward model

Approach to question #3

- Full Waveform Inversion (FWI): reconstructs the earth's subsurface properties from local measurements of a seismic wavefield
- Minimise the "misfit" between numerical predicted and physically recorded data

$$c^* = \text{argmin}_c f(c); \quad f(c) = \frac{1}{2} \sum_s \|R_s u_s(c) - d_s\|^2$$

- The optimization cycle:
  1. Make a guess of the subsurface properties ("the model")
  2. Solve the wave equation
  3. Compute predicted data
  4. Compare with the observed data
  5. Compute misfit and update the model
  6. Go back to [2.]
- Problem: FWI assumes fixed position of sensors

Approach to question #3

- Our approach: Generalize FWI so that it incorporates different position of sensors.
- Minimise the "misfit" between different models:

$$p^* = \text{argmin}_p \Psi(p); \quad \Psi(p) = \frac{1}{2}\|W(d(p)) - \tilde{c}\|^2$$

where

- $\tilde{c}$ is a discrete fixed model
- $u = u(c)$ is the solution of the forward model
- $\Gamma(p)$ is the observation operator at positions p
- $d(p) = \{\Gamma(p)u\}$ compute predicted data at points p
- $W(d) = c^*$, the FWI of d, i.e.

$$W(d) = c^* = \text{argmin}_c f(c)$$

Approach to question #3

Possible algorithm

1. Fix a discrete model for c
2. Choose an initial configuration for the sensors
3. Compute the solution of the forward problem
4. Compare computed data at position p with real data
5. Find a model c^* that minimizes [4.]
6. Compare this model with the initial model in [1.] and find position of sensors that reduce the error
7. Go back to [3.]

Approach to question #3

Current progress:

- We can compute the gradient of Ψ
- This involves inversion of special Jacobian matrices which is expensive, we considered cheap approximations using random sampling
- Further investigation is necessary

Approach to question #3

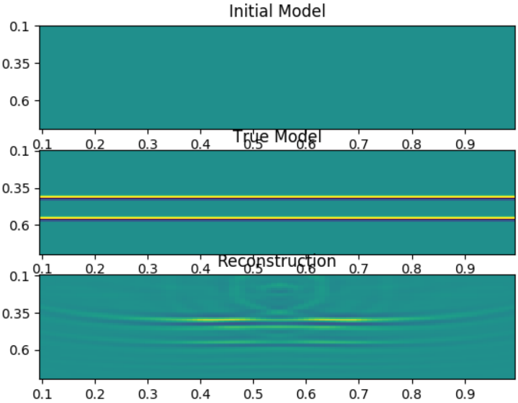

Figure: Ten receivers evenly spread[1]

[1] pysit code from the Imaging and Computing Group at MIT

Approach to question #3

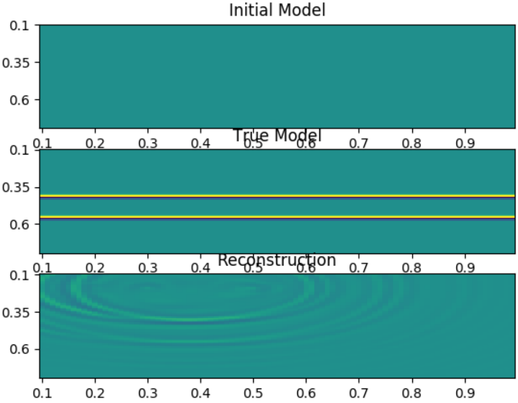

Figure: Two receivers positioned at 0.2 and 0.3[1]

[1] pysit code from the Imaging and Computing Group at MIT

Approach to question #3

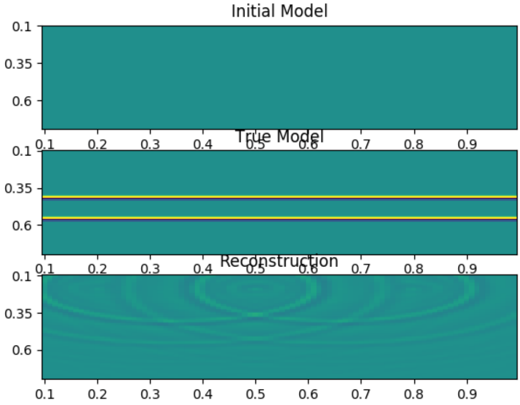

Figure: Two receivers positioned at 0.2 and 0.8[1]

[1] pysit code from the Imaging and Computing Group at MIT

Approach to question #3

Outcome of the optimization process with Nelder Mead

- Start: $[0.1, 0.8]$, optimized: $[0.196, 0.801]$
- Start: $[0.1, 0.6]$, optimized: $[0.125, 0.608]$
- Start: $[0.1, 0.2]$, optimized: $[0.112, 0.201]$

ITT 6 outcome

- Value of gradient measure: Can we justify the benefit of gradient measures? ✓
- What is a good basis to
  - represent the data
  - represent the image
  - to compute the wave propagation
    Necessary conditions found, exact basis function not computed, still in progress
- Model design: What is the difference in designing the problem with respect to the data or the image?
  Treated in approach to question #3, we developed a theoretical basis to answer the question of optimal sensor position, work still in progress

ITT 6 outcome

- Value of gradient measure: Can we justify the benefit of gradient measures? ✓
- What is a good basis to
  - represent the data
  - represent the image
  - to compute the wave propagation

    Necessary conditions found, exact basis function not computed, still in progress
- Model design: What is the difference in designing the problem with respect to the data or the image?

  Treated in approach to question #3, we developed a theoretical basis of to answer the question of optimal sensor position, work still in progress

Thank you for your attention