INFERENCE AND FILTERING FOR OIL DRILLING

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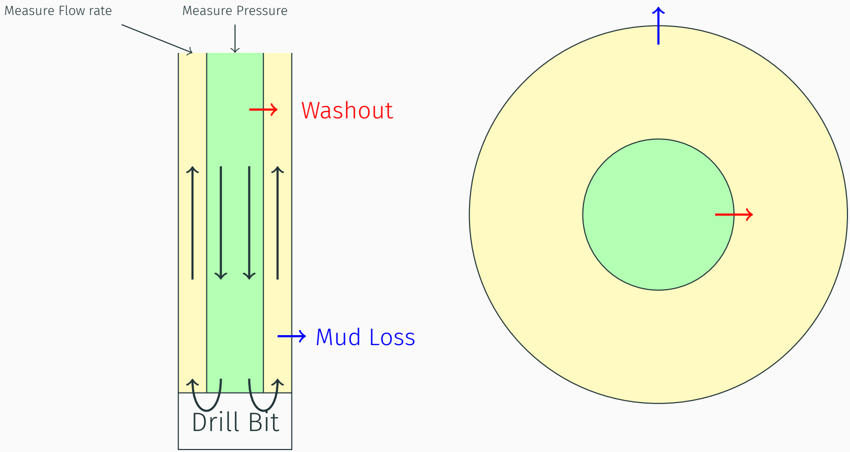
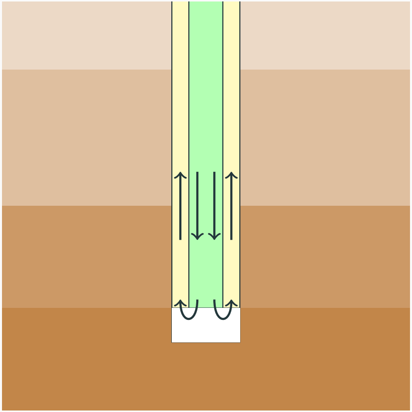
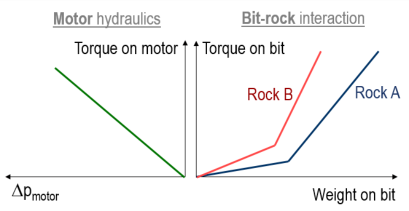


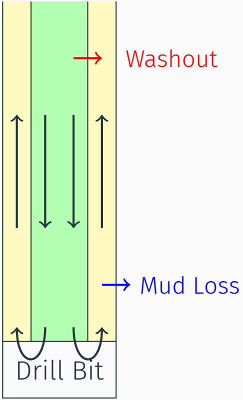
Problem:

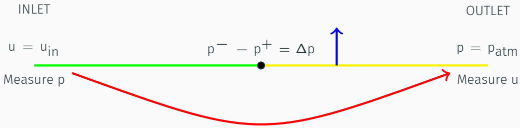
Find washout and mud loss parameters from measurements at inlet/outlet.





$$T = \gamma \Delta p_m = \begin{cases} \alpha_1 W_B & W_B < \omega \\ \alpha_2 W_B + \beta_2 & W_B \geq \omega \end{cases}$$





$$\frac{\partial A\rho}{\partial t} + \frac{\partial A\rho u}{\partial z} = s_m A$$

$$\frac{\partial A\rho u}{\partial t} + \frac{\partial A\rho u^2}{\partial z} = -A\frac{\partial p}{\partial z} + A\left(\left(\frac{\partial p}{\partial z}\right)_f + \rho g_z\right)$$

where $p \propto \rho$.
Source s_m models mud loss and washout.

Ideally, we should infer in real-time:

1. Mud loss parameters $k, p_L, t_{0,l}, z_m$
2. Washout parameters $K, t_{0,w}, z_w$
3. Rock/drill bit parameters $\{\alpha_{1,i}, \alpha_{2,i}, \omega_i\}_{i=1}^{n_{rock}}, i(t)$

INVERSE PROBLEM

Ideally, we should infer in real-time:

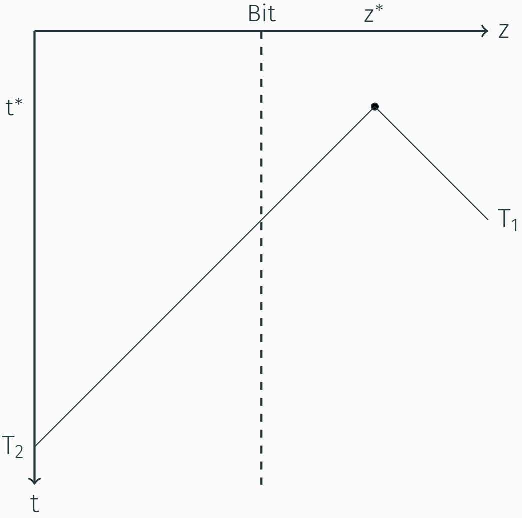
1. Mud loss parameters $k, p_L, t_{0,l}, z_m$
2. Washout parameters $K, t_{0,w}, z_w$

(items 1–2) - Smoothing Methods

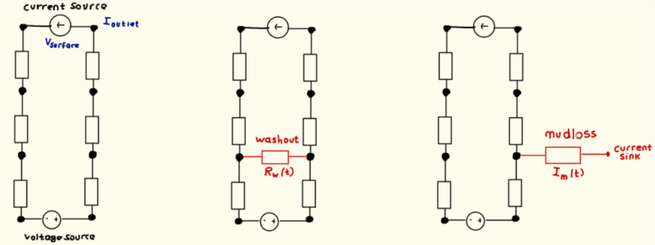
3. Rock/drill bit parameters $\{\alpha_{1,i}, \alpha_{2,i}, \omega_i\}_{i=1}^{n_{rock}}, i(t)$ - Filtering Methods

Ultimate aim is to combine smoothing and filtering...

INFORMATION FROM MEASUREMENT LAG



TOY MODEL: TELEGRAPHER'S EQUATIONS



Telegrapher's equations model voltage (pressure) and current (momentum) in a wire:

$$\frac{\partial V}{\partial t} + \frac{\partial I}{\partial z} = 0$$
$$\frac{\partial I}{\partial t} + \frac{\partial V}{\partial z} = 0$$

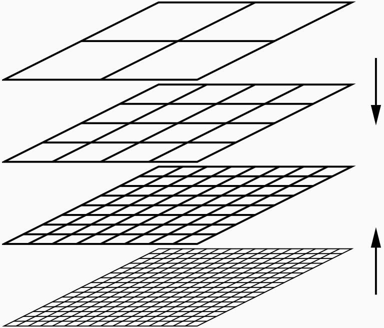
Observations can be mapped directly to mud loss parameters ϕ_*, t_*.

E.g. 4D-VAR to estimate mud loss parameters $\theta = (z_m, k, t_0)^T$; Optimize

$$
\begin{aligned}
J(\boldsymbol{\theta}_a) = (\boldsymbol{\theta}_a - \boldsymbol{\theta}_f)^T B^{-1} (\boldsymbol{\theta}_a - \boldsymbol{\theta}_f) + (H(\boldsymbol{\theta}_a) - \mathbf{y}_1)^T R^{-1} (H(\boldsymbol{\theta}_a) - \mathbf{y}_1) \\
+ (H(M(\boldsymbol{\theta}_a)) - \mathbf{y}_2)^T R^{-1} (H(M(\boldsymbol{\theta}_a)) - \mathbf{y}_2)
\end{aligned}
$$

where B and R are forecast and observation error covariance matrices respectively - these can be estimated from Kalman filtering.

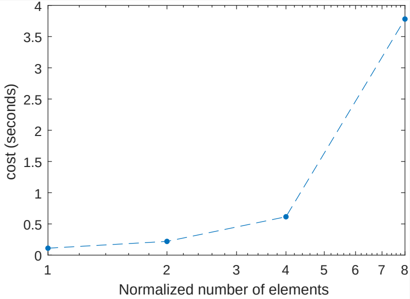
- Gradient evaluation requires adjoint model (linearization - work to be done)
- Ensemble 4D-VAR gives uncertainty estimate.

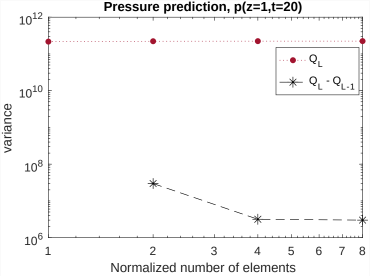


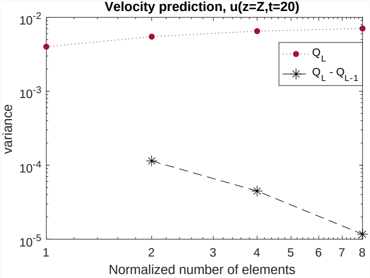
Multilevel methods reduce variance in estimates while costing less than the equivalent number of fine solves.

Filtering method: Multilevel Sequential Monte Carlo.

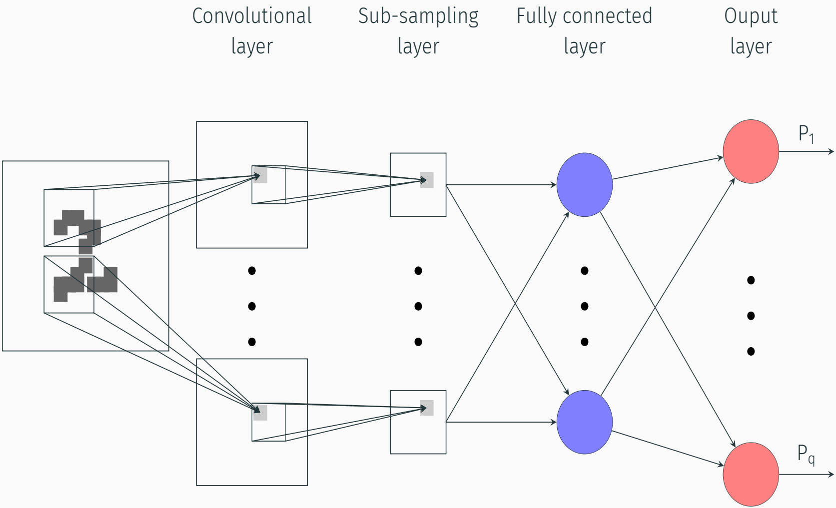
MULTILEVEL METHODS: SUITABILITY



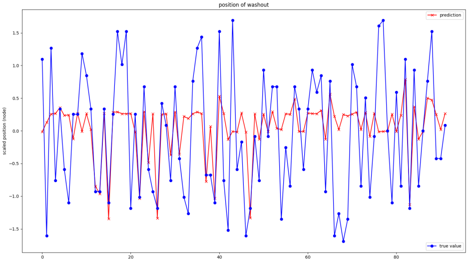


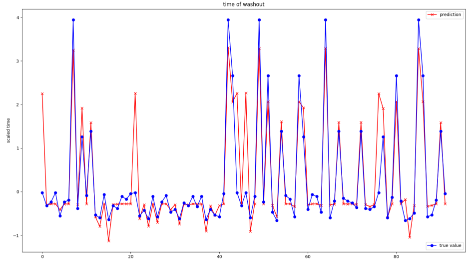


NEURAL NETWORK APPROACH



NEURAL NETWORK APPROACH: RESULTS



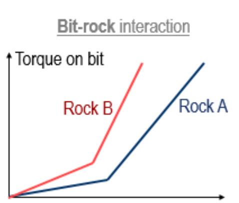


FILTERING

Objective: Rapidly and accurately detect a washout or mud loss. We need to estimate the parameters $\theta = [\alpha_1, \alpha_2, \omega]$ using a data driven method.

In other words: not taking into account mud loss and washout but infer the parameters of the models using online data.

Idea: using Unscented Kalman Filtering: more robust method when the function is non linear.



Drill bit and rock interaction

$$T = \begin{cases} \alpha_1 W_\mathrm{B} & \text{if } W_\mathrm{B} < \omega \\ \alpha_2 W_\mathrm{B} + \beta_2 & \text{if } W_\mathrm{B} \geq \omega \end{cases}$$

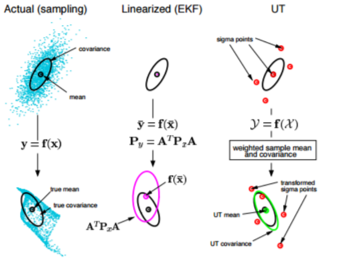


Figure 1: Example of the UT for mean and covariance propagation. a) actual, b) first-order linearization (EKF), c) UT.

$$
\begin{aligned}
\mathcal{X}_0 &= \bar{\mathbf{x}} && && (15)\\
\mathcal{X}_i &= \bar{\mathbf{x}} + \left(\sqrt{(L+\lambda)\mathbf{P}_{\mathbf{x}}}\right)_i && i = 1,\dots,L\\
\mathcal{X}_i &= \bar{\mathbf{x}} - \left(\sqrt{(L+\lambda)\mathbf{P}_{\mathbf{x}}}\right)_{i-L} && i = L+1,\dots,2L\\
W_0^{(m)} &= \lambda/(L+\lambda)\\
W_0^{(c)} &= \lambda/(L+\lambda) + (1-\alpha^2+\beta)\\
W_i^{(m)} &= W_i^{(c)} = 1/\{2(L+\lambda)\} && i = 1,\dots,2L
\end{aligned}
$$

UNSCENTED KALMAN FILTERING: ALGORITHM

Time update:

$$\boldsymbol{\mathcal{X}}^x_{k|k-1} = \mathbf{F}[\boldsymbol{\mathcal{X}}^x_{k-1}, \boldsymbol{\mathcal{X}}^v_{k-1}]$$

$$\hat{\mathbf{x}}^-_k = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{X}^x_{i,k|k-1}$$

$$\mathbf{P}^-_k = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{X}^x_{i,k|k-1} - \hat{\mathbf{x}}^-_k][\mathcal{X}^x_{i,k|k-1} - \hat{\mathbf{x}}^-_k]^T$$

$$\boldsymbol{\mathcal{Y}}_{k|k-1} = \mathbf{H}[\boldsymbol{\mathcal{X}}^x_{k|k-1}, \boldsymbol{\mathcal{X}}^n_{k-1}]$$

$$\hat{\mathbf{y}}^-_k = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Y}_{i,k|k-1}$$

Measurement update equations:

$$\mathbf{P}_{\tilde{\mathbf{y}}_k \tilde{\mathbf{y}}_k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}^-_k][\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}^-_k]^T$$

$$\mathbf{P}_{\mathbf{x}_k \mathbf{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{X}_{i,k|k-1} - \hat{\mathbf{x}}^-_k][\mathcal{Y}_{i,k|k-1} - \hat{\mathbf{y}}^-_k]^T$$

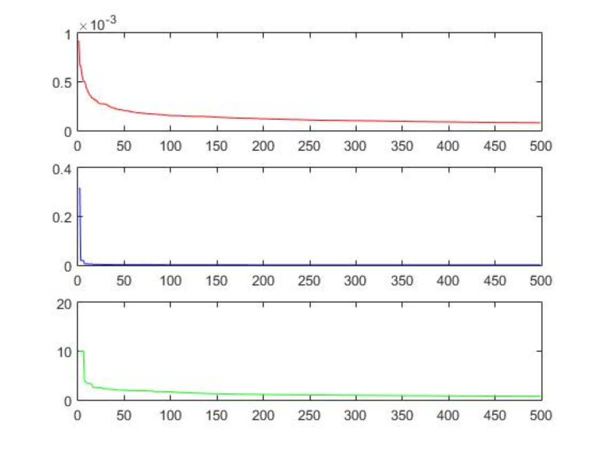
$$\mathcal{K} = \mathbf{P}_{\mathbf{x}_k \mathbf{y}_k} \mathbf{P}^{-1}_{\tilde{\mathbf{y}}_k \tilde{\mathbf{y}}_k}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}^-_k + \mathcal{K}(\mathbf{y}_k - \hat{\mathbf{y}}^-_k)$$

$$\mathbf{P}_k = \mathbf{P}^-_k - \mathcal{K} \mathbf{P}_{\tilde{\mathbf{y}}_k \tilde{\mathbf{y}}_k} \mathcal{K}^T$$

Assumptions:

- only one type of rock
- the function H has been approximated.



Problems: How to be sure that the parameters will change when the rock changes. Methods: Use a quality index:$(y_t - y_o b)P_y^{-1}(y_t - y_o b)'$ or least square method to decide which model fits best the data.
First idea: Use for each model the UKF algorithm and decide which model to use using the quality index.
Second idea: We will always consider two models $\theta_1 = [\alpha_1, \alpha_2, \omega]$ with a covariance matrice P_θ
and another model with a larger covariance.
We will only update the first model. When the first model is not good enough we will decide to use the second model.

Thanks for listening!