Microbubble Modelling

June 9, 2017

Problem Context

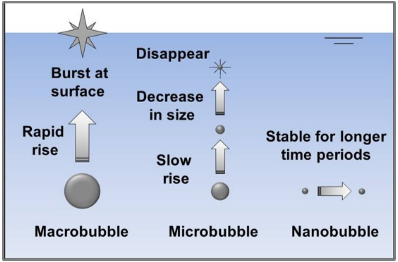

Figure: Comparison of bubble size dynamics.

Figure: Waste water aeration.

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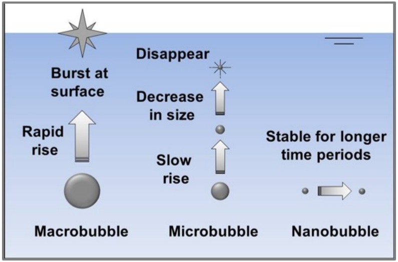

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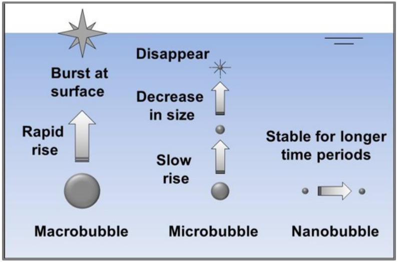

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Figure: Waste water aeration.

- Seek to aerate waste water by introducing bubbles of air into the water.
- Efficiency of treatment process is determined by mass transfer.

Motivation for Using Microbubbles

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However:

- **Bubbles too large** means a gas loss due to bubble rise.
- **Bubble too small** means the bubbles do not rise and the gas is only distributed at the bottom of the tank.

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Overall Aim

To efficiently transfer mass from the gas phase in the bubbles to the liquid water phase.

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- We seek to do this, initially, by modelling and understanding the behaviour and the dynamics of a single bubble.

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- Position change (vertically)
- Mass transfer

Non - Dimensional Model Equations

Rayleigh - Plesset

$$\ddot{R} = \frac{1}{\hat{\rho}_\ell}\Bigg(\underbrace{\hat{R_g} T \frac{3 m_g}{4\pi R^3}}_{=:P_b} \underbrace{- \hat{P}_{\text{atm}} - \hat{g}\hat{\rho}_\ell z}_{=:-P_\infty}\Bigg) - \frac{3}{2}\frac{\dot{R}^2}{R} - \frac{4\hat{\nu}_\ell \dot{R}}{R^2} - \frac{2\hat{\gamma}}{\hat{\rho}_\ell R^2}$$

Equation of Motion

$$\ddot{z} = \underbrace{-\frac{\dot{z}\dot{m}_g}{m_g}}_{\text{Momentum Change}} \underbrace{- \frac{\frac{4\pi}{3} R^3 \hat{\rho}_\ell - m_g}{m_g}\hat{g}}_{\text{Buoyancy}} \underbrace{- 6\pi C \hat{\nu}_\ell \hat{\rho}_\ell \frac{\dot{z} R}{m_g}}_{\text{Stokes Drag}}$$

Mass Transfer

$$\dot{m}_g = 3 n_* \frac{m_g}{R}$$

First Results

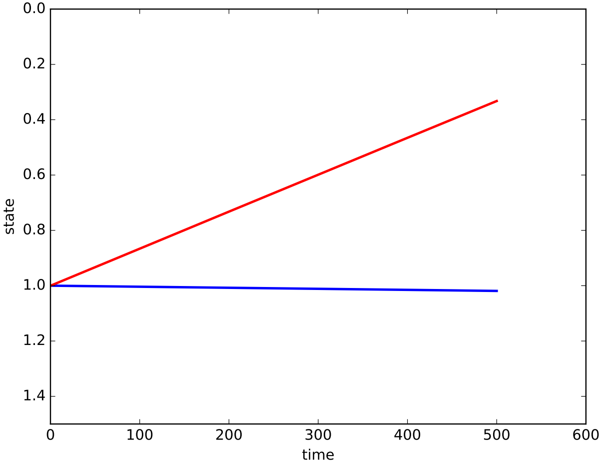

Figure: Stable bubble radius. *Red* line is relative distance from surface in z and *blue* line is the relative bubble radius.

Some Problems

Characteristic Values

$$\bar{R} = 5\mu\text{m}, \quad \bar{v} = u_\infty = 5.5 \times 10^{-3}\text{ms}^{-1}, \quad \bar{\rho} = \rho_\ell$$

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- Assume Stokes' Law holds

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- Food processing.

Research Outline